A century later: teaching Quantum Mechanics

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Almost a century ago…

1924: Born used* (but not coined**) the term "Quantum Mechanics (QM)" for indicating a precise theory that would replace the Old Quantum Theory (OQT).

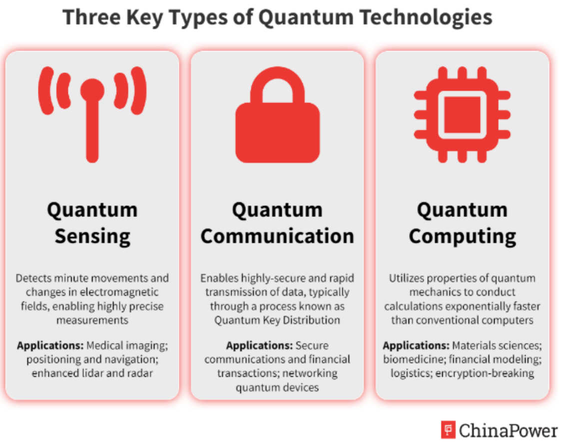
1925: Heisenberg's "apocalypse" in Heligoland***.

* Born M (1924) Z. Phys. 26(1): 379-395.
** Already used in 1921 and 1922…but with a different meaning. For details, see: Kragh H (2024) The Names of Science. Oxford: Oxford University Press.
*** Heisenberg W (1925) Z. Phys. 33(1): 879-893.

...and still now

<u>Second quantum revolution</u>.

QM technological application: immense potential, incredible versatility, and extraordinary power (also concerning energy issues).



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Need for a cultural perception of QM

Significance and utility of QM: not limited to technological application, but intrinsic importance.

Goal: a cultural heritage for everyone:

*"Quantum physics is no longer an abstract theory for specialists. We must now absolutely include it in our education and also in our culture."**

* Cohen-Tannoudji C (2008) Honeywell-Nobel Laureate Lecture Series.

Need for a cultural perception of QM

Time that QM becomes part of people's knowledge

Starting point: student's education.

Need and urgency to reconstruct the crucial role of physics as a cultural promoter of scientific knowledge in society*.

Cultural understanding of QM: even more considerable and immediate efforts, in a society where the application of physical knowledge risks overshadowing the true essence of physics.

* GIREP thematic group "Cultural Understanding of Physics: instruments and methods" (https://www.girep.org/thematic-groups/cultural-understanding-of-physics-instruments-and-methods/)

The research project of Milan PERG

Aim of Milan PERG: give QM a central role in PE, by trying to outline an effective didactic reconstruction of such a theory (with its historical development).

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Coherent and detailed path for introducing physical and mathematical foundation of QM.

Goal: design, on the model of the educational reconstruction, a didactic approach for teaching QM at high school and in non-physics degrees courses → a significant and formative itinerary as well as a cultural framework*.

* Lovisetti L, Organtini G, Giliberti M (2023) Il Nuovo Cimento C 46(6): 200.
Lovisetti L, Melli E, Giliberti M (2024) J. Phys.: Conf. Ser. 2750: 012022.
Lovisetti L (2024) Il Nuovo Cimento C 47(5): 277.

The international landscape

Importance of understanding Quantum Physics (QP) for the education of citizens.

Since the early 2000s: reforms in educational systems, teaching of QP mandatory (OQT and/or QM) → over 1,700 articles published in 2000-2024.

All over the world, different approaches*:

1) Feynman's path-integral → visual approach and few mathematics.
2) Matrix approach starting from spin ("spin-first") → quantum state and superposition.
3) Linear systems → superposition and linear operators.
4) More recently, QM through two-level system (qubits).

* A comprehensive overview can be found in: Michelini M, Stefanel A (2021) In B. Jarosievitz and C. Sükösd (eds) Teaching-Learning Contemporary Physics. Springer: Cham, pp 3-17.

An axiomatic structure for QM

Well-structured approaches, providing QM concepts…but no explanation of how postulates are formulated!

In Milan: QM together with its postulates*, induced from the phenomenology, and used to explain the real world (well-potentials, atoms…).

Meaning of a theory = complete set of mathematical formalism + field of applicability + correspondence rules.

Idea: theoretical and formal construction of QM, explaining why a specific mathematical structure is obtained from the phenomenology (why a linear theory, Hilbert spaces, operators, a scalar product…), to induce a set of principles and justify all the postulates.

* Lovisetti L, Organtini G, Giliberti M (2023) Il Nuovo Cimento C 46(6): 200.
Lovisetti L, Melli E, Giliberti M (2024) J. Phys.: Conf. Ser. 2750 012022.

A historical glance: QM but also OQT

*“There is a tendency nowadays for undergraduates to learn their physics completely from textbooks, not becoming acquainted with the original literature and therefore not realizing how the subject grew and developed.”**

General idea: historical aspects are not useful (excessive time and qualitative presentation)**.

In Milan: studying also the OQT and historical aspects of QM is beneficial***.

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Physics curricula intertwined with aspects related to the Nature of Science (NOS).

* Haar D ter (1966) The Old Quantum Theory. Pergamon Press Inc., New York, backcover.
** Michelini M, Stefanel A (2021) In B. Jarosievitz and C. Sükösd (eds) Teaching-Learning Contemporary Physics. Springer: Cham, pp 3-17.
*** Lovisetti L (2024) Il Nuovo Cimento C 47(5): 277.

A historical glance: QM but also OQT

OQT+QM and physics contents + historical perspectives → incremental evolution of scientific knowledge.

OQT → dynamic interplay between theoretical predictions and experimental verification.

OQT anomalies and limitations → development of QM framework (provisional scientific theories).

OQT and radical changes to QM → Kuhn's idea of paradigm shift.

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A meaningful framework for an approach that leverages active teaching/learning sequences*.

*Lovisetti L (2024) Il Nuovo Cimento C 47(5): 277.

Historical and conceptual issues

Step 1: textbooks call "QM" but is OQT.

OQT: not a real theory, only a collection of some *ad hoc* models, made for explaining specific phenomena.

\+

Traditional presentations: historical and conceptual inaccuracies misinterpretations.

"The problem of the stability of the hydrogen atom was solved by Schrödinger."

How could it, since the (Coulomb) Hamiltonian of the atom does not include emission terms?

"Planck was the first in quantizing energy."

Maybe the energy of the harmonic oscillators…

"Einstein's 1905 quantized model was based on and naturally extended Planck's idea."

Well, actually, it was not…

"In Bohr's hydrogen atom, the energy levels were quantized."

Actually, they were discretized, not quantized…

Historical and conceptual issues

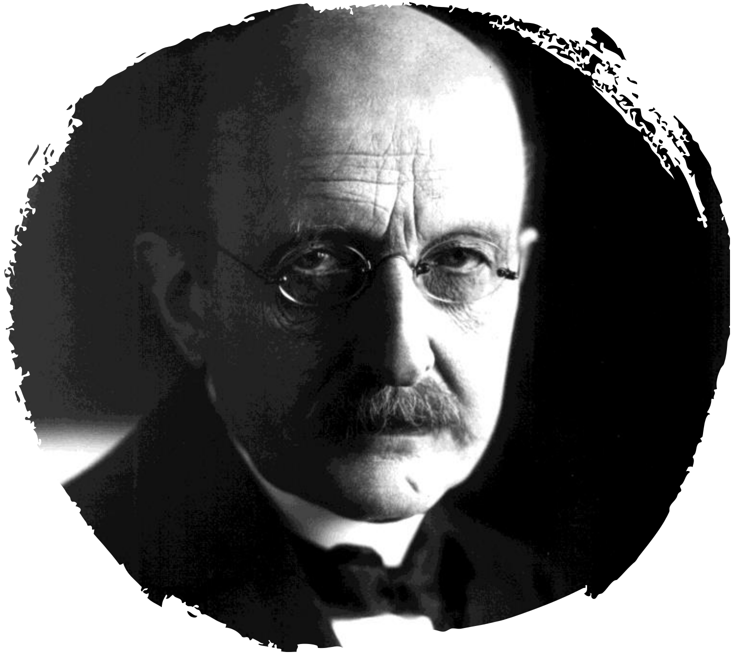
"Planck was the first in quantizing energy."

In 1900-1901*, he quantized the energy of the harmonic oscillators in the black (energy as multiple of $h\nu$).

$$u(\nu, T)d\nu = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{\frac{h\nu}{T}} - 1}d\nu$$

Radiation distributed continuously.

* Planck M (1900) Ann. Phys. 306(1): 69-122.
Planck M (1900) Verh. Dtsch. Phys. Ges. 17: 237-245.
Planck M (1901) Ann. Phys. 309(3): 553- 563.



Max Planck

Historical and conceptual issues

"Einstein's 1905 quantized model was based on and naturally extended Planck's idea."

In 1905*, Einstein applied quantization to the radiation itself (light). The energy is that of radiation and travels in packets: it is not continuous!

Einstein: light quanta as real physical entities, and not merely a mathematical device → a fundamental change in the understanding of the nature of light.

Albert Einstein

* Einstein A (1905) Ann. Phys. 17(6): 132-147.
Kragh H (1992) Sci. Educ. 1: 349-363.

Historical and conceptual issues

"In Bohr's hydrogen atom, the energy levels were quantized."

The term "discretized" is more appropriate: in fact, in Bohr's trilogy of 1913*, the separation between the energy levels is not a multiple of a fundamental quantity…

$$E_n = \frac{me^4}{8\varepsilon_0^2 h^2 n^2}$$

* Bohr N (1913) London Edinburgh Philos. Mag. & J. Sci. 26(151): 1-24.
  Bohr N (1913) London Edinburgh Philos. Mag. & J. Sci. 26(153): 476-502.
  Bohr N (1913) London Edinburgh Philos. Mag. & J. Sci. 26(155): 857-875.

Niels Bohr

Historical and conceptual issues

“The problem of the stability of the hydrogen atom was solved by Schrödinger.”

How could it, since, in Schrödinger’s formula, the (Coulomb) Hamiltonian of the atom does not include emission terms?

$$H = \frac{\hbar^2}{2me^2}\nabla^2 - \frac{e^2}{4\pi\varepsilon_0 r}$$

* Schrödinger E (1926) Ann. Phys. 384(4): 361-376.
Schrödinger E (1926) Ann. Phys. 384(6): 489-527.
Schrödinger E (1926) Ann. Phys. 385(13): 437-490.
Schrödinger E (1926) Ann. Phys. 386(18): 109-139.

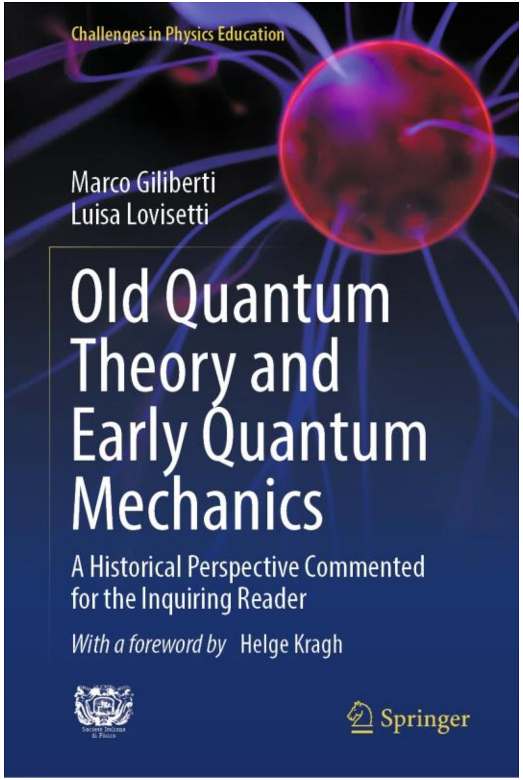
Erwin Schrödinger

OQT and MQ for inquiring minds

Historical reconstruction of OQT and early QM (800 primary sources and 300 secondary sources): a book*.

Historical account enriched with pedagogical explanations and comments for “inquiring minds”.

From a didactic standpoint: this part (especially OQT) provided a part of the phenomenological basis from which some fundamental principles could be formulated.



* Giliberti M, Lovisetti L (2024) Old Quantum Theory and Early Quantum Mechanics. A Historical Perspective Commented for the Enquiring Reader. Springer: Cham.

The principles (from OQT + experiments)

Let us focus on a specific example…

Principle I: Same formulas: the formulas used for physical quantities with classical analogues are formally identical to those in classical physics.

- Planck: the black-body spectrum (energy, harmonic motion…)*,
- Einstein: photoelectric effect**,
- Bohr: hydrogen atom (energy, angular momentum, electron velocity…)***,
- Franck-Hertz experiment****.

This is not a trivial fact! In relativity it does not happen…!

* Planck M (1900) Verh. Dtsch. Phys. Ges. 17: 237-245.
** Einstein A (1905) Ann. Phys. 17(6): 132-148.
*** Bohr N (1913) London Edinburgh Philos. Mag. & J. Sci. 26(151): 1-24; 26(153): 476-502; 26(155): 857-875.
**** Franck J, Hertz G (1914) Verh. Dtsch. Phys. Ges. 16: 457-467.

Axiomatic reconstruction of QM

Step 2: exploit selected historical elements + experiments as a starting point and as a guide → reconstructing QM for didactic purposes.

Justify three postulates (~ von Neumann* and proposed by most physics textbooks).

1. To every physical system, a Hilbert space $\mathcal{H}$ is associated. Every state of the system is represented by a unit vector $|\psi\rangle$ in $\mathcal{H}$. The temporal evolution of this state vector is governed by an equation of the form $i\hbar\frac{d|\psi\rangle}{dt} = \hat{H}|\psi\rangle$, where $\hat{H}$ is a suitable self-adjoint operator.
2. To every observable quantity G, there corresponds a self-adjoint operator $\hat{G}$ on $\mathcal{H}$. The different eigenvalues g_k of $\hat{G}$ represent the possible results of measuring the quantity G. If $|g_k\rangle$ is the normalized eigenvector corresponding to the eigenvalue g_k, and the state of our system is given by $|\psi\rangle$, then the probability that a measurement performed on the system yields the result g_k is given by (Born's rule): $p(g_k) = |\langle g_k|\psi\rangle|^2$.
3. The state of the physical system, immediately after a measurement of the quantity G yields the result g_k, is given by $|g_k\rangle$.

* von Neumann J (1932) Mathematische Grundlagen der Quantenmechanik. Berlin: Springer.

Trials organised by Milan PERG

6 experimentations (online/in presence) with 320 high school students and 241 teachers:

- 2021/22: “The Elegance of Quantum Mechanics” (PLS): 15h, only QM.
- 2022/23: “Old (but Gold) Quantum Theory” (PNRR): 15h, only OQT.
- 2022/23: “The Elegance of Quantum Mechanics II” (PON, at school): 30h, only QM
- 2023/24: “Old (but Gold) Quantum Theory II” (PNRR): 15h, only OQT.
- 2023/24: “Principles and Equations of Physics III: Quantum Mechanics” (PLS): 8h OQT + 22h QM.
- 2024/2025: “Introduction to Quantum Mechanics” (PLS): 10h OQT + 20h QM (only teachers)

Students + teachers: evaluate effectiveness and assessment of comprehension.

3 experimentations with 59 master’s degree students (mathematics or physics): 40h.

During the meetings

- Experimental activities (thermal camera, spectral lamps, measurement of h...),
- 10-minute workgroups and commented reading of original papers (*e.g.*, Thomson's paper of 1904, EPR's and Bohr's papers of 1935),
- qualitative examples (*e.g.*, Einstein's idea of light-quanta and its heuristic model),
- quantitative aspects (*e.g.*, calculation of Planck's unit of measure, determination of eigenvalues or eigenvectors),
- open questions typically not covered in textbooks (*e.g.*, *«Why in the Compton effect does an electron not "eat" a quantum of light as in the photoelectric effect?»*),
- interactive graphic examples created with GeoGebra and a custom-built web app,
- quizzes using Kahoot!,
- interactive simulations with Quantum Flytrap.

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During the meetings

Conceptual enquiry approach on theoretical tasks (active reading and comprehension).

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Physics contents in details, in a meaningful and coherent way.

Aim: create a groundwork for fostering critical thinking skills as well as providing a cultural and reasoned perspective on QP, highlighting also the crucial disciplinary and learning knots for both OQT and QM.

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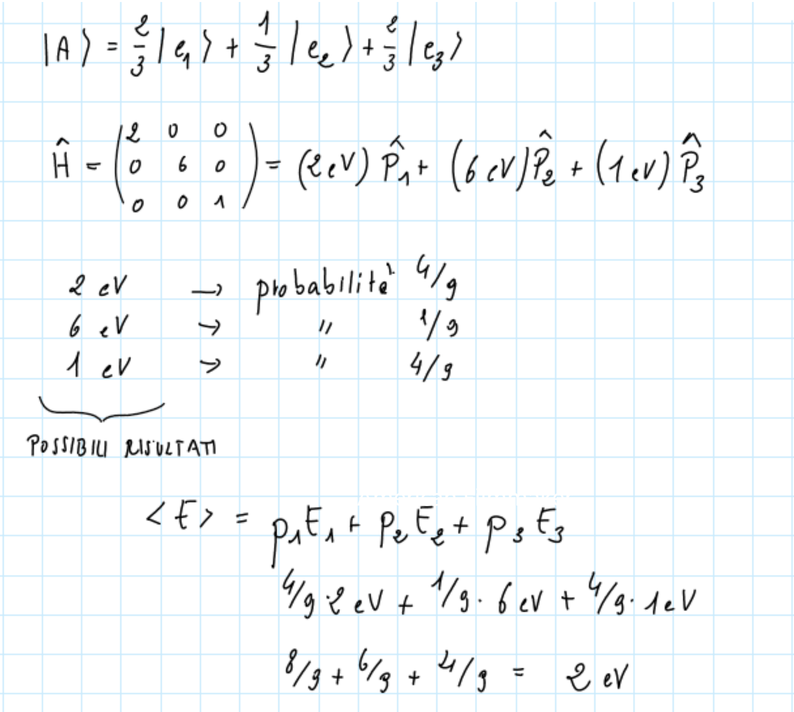
During the meetings

"Suppose you want to know the energy of an electron in an atom. You know that:

1. the state it is in, before carrying out the measurement, is: $\hat{A} = \frac{2}{3}|e_1\rangle + \frac{1}{3}|e_2\rangle + \frac{2}{3}|e_3\rangle$.
2. the operator associated with the energy variable is: $\hat{H} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

Determine: a) how many and what are the possible results of this measure; b) with what probability you get one of the possible outcomes; c) the average energy value $< E >$."



A correct example of students' answers.

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Results and effectiveness

Through different data sources	Participants	Control group
Average grade	6.1 (SD 0.7)	6.0 (SD 0.9)
Ability in linking ideas in a given context (with the Knowledge Integration Construct)**	68%	15%
Level of critical thinking	71%	33%
Solving new problems in situations never encountered before	72%	29%
Awareness of NOS (models, theories, formalism…)	81%	23%

Control group: 86 undergraduate students majoring in mathematics or physics, along with 46 teachers (traditional approach to QP).

* Liu OL, et al (2008) Educ. Assess. 13: 33-55.

Conceptual and learning knots: OQT

Great difficulties in exploiting/using the OQT physical models to elucidate diverse scenarios and make predictions.

Rely on other pre-existing knowledge, inaccurate analogies with familiar concepts (43%).

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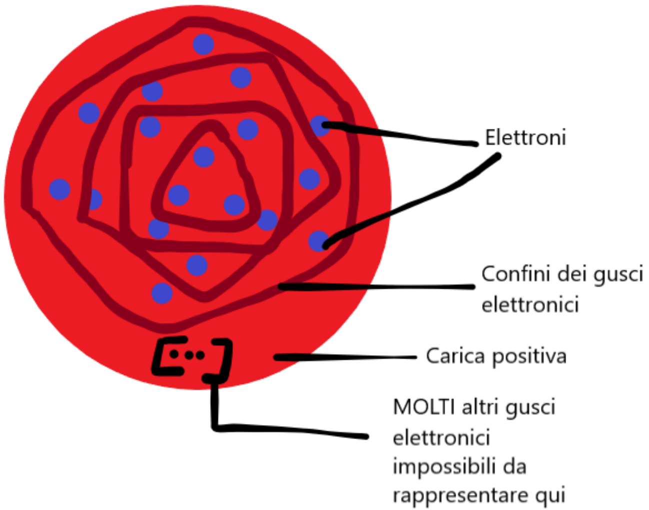
Deficiency in the capacity to thoroughly explore the model (lack of profound conceptual understanding).

Conceptual and learning knots: OQT

Deep dichotomy between verbal descriptions and mental images; e.g. atomic models: Thomson (1904), Bohr (1913).

Q: "Describe Thomson's atomic model".
→ 74% correct (regular and precise geometrical structure), 24% "plum pudding".

Q: "Draw a carbon atom using Thomson's model" .
→ 36% correct, 41% "plum pudding", 19% octet rule (1916), 8% protons (1919) and/or neutrons (1932) or other "anachronistic" elements.



A correct example of students' drawings of Thomson's model.

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Conceptual and learning knots: QM

- Difficulties in visualizing spaces with more than 3 dimensions (41%);
- difficulties in the concept of self-adjoint operator (26%);
- confusion between states and operators (29%) and between the operation of measurement with the action of the operator on the state (63%);
- confusion between the trajectory of the state vector in Hilbert space and the trajectory of the particle in physical space (37%).

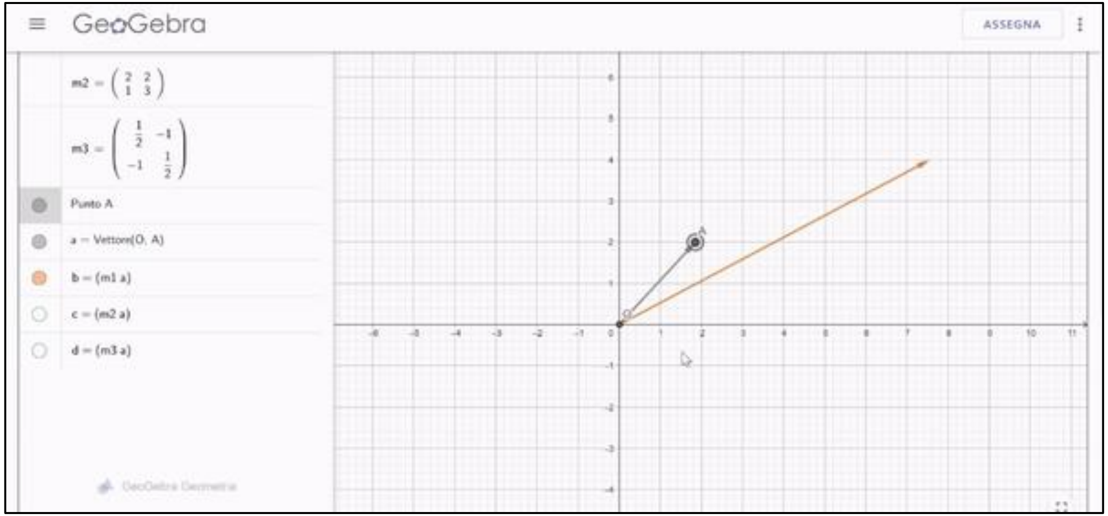
However…

…the mathematical and formal aspects (complex linear spaces, matrices, Dirac notation…) not seen as obstacles, but rather as something reassuring and helpful.

Conceptual and learning knots: QM

GeoGebra + web-app (manipulate vectors and matrices): allowed to overcome some difficulties (*i.e.*, for eigenvalues and eigenvectors).

23%: vectors with same direction but rotated of 180° as different eigenvectors.



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Visualization in both OQT and QM

Aspect of visualization: emerged as a crucial point.

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Important to adopt multi-modal and multi-focal approaches + tools that allow for the visualization and manipulation of aspects of QP.

Increasing in the explicit request for a "pictorial" rendering and imaginative representation of models of OQT.

Open horizons and unfinished quests

Some aspects still need to be addressed and/or refined in future research.

Priorities (special attention):

- development of a sub-section of the path focused on the concept of spin,
- development of specifically-designed and didactically meaningful experimental or multimedia tools.

*"But that's another story and shall be told another time..."**

* Ende M (2014) The Neverending Story. London: Puffin Books, p 511.

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