



NICOLAUS COPERNICUS  
UNIVERSITY  
IN TORUŃ

# Materials design from first principles: minimizing energy

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**Uppsala University, Sweden**



## Humanity's interest in extreme phenomena

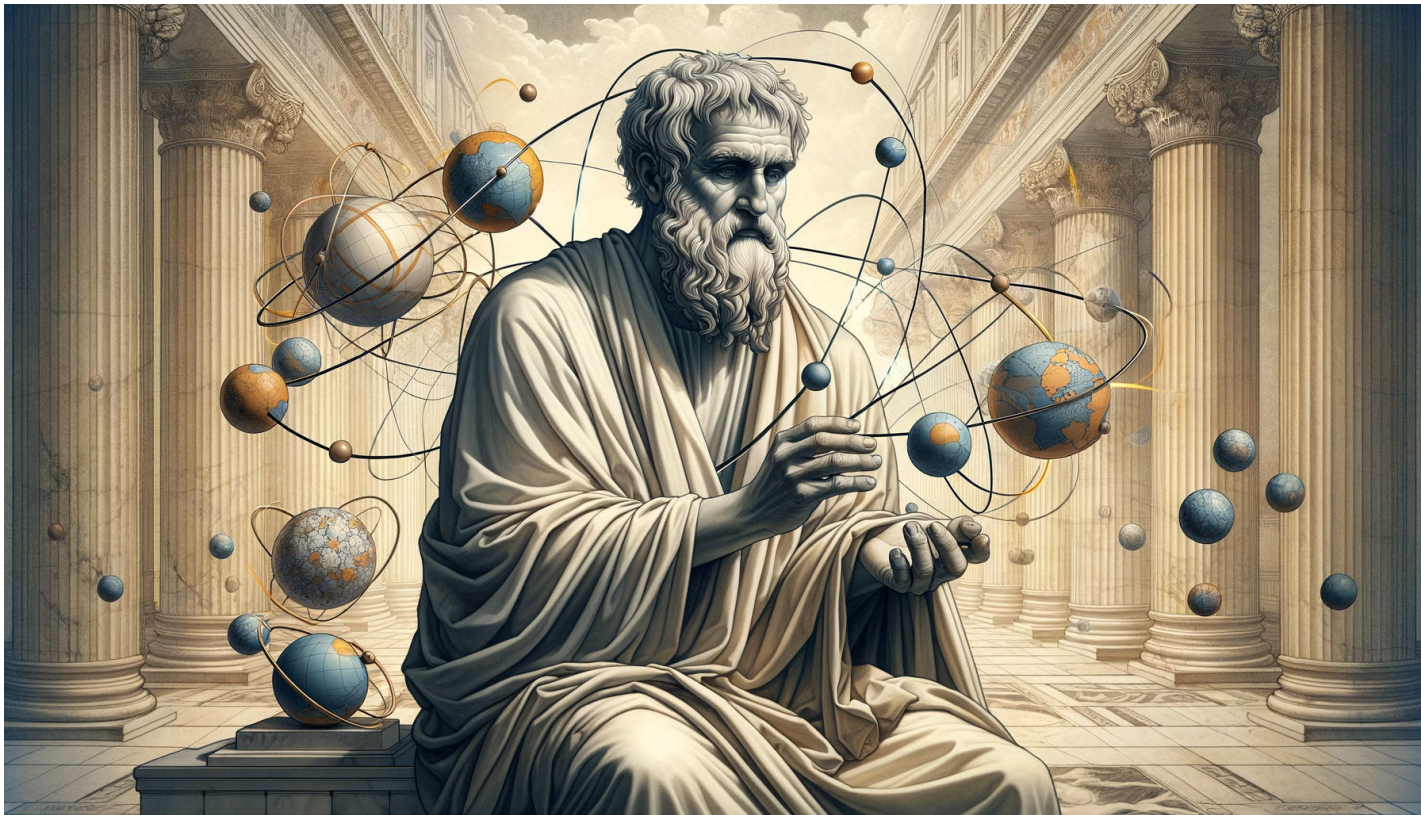


Man lives between the infinitely large and the infinitely small.

~ Blaise Pascal

AZ QUOTES

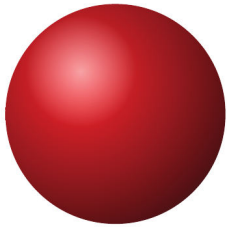
## Democritus and the rise of the atomistic model



from [academichelp.net](https://www.academichelp.net)

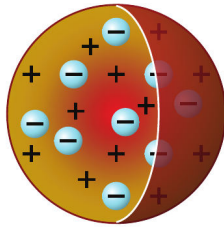
# A long journey towards our current understanding

An atom can be modeled as a nucleus with a system of interacting electrons



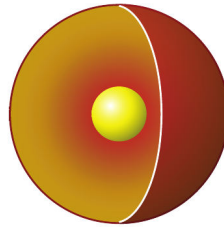
**Atom model**

Dalton (1803)



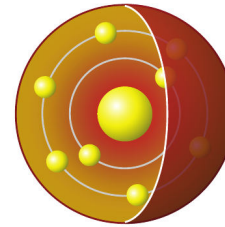
**Pudding  
atom model**

Thomson (1904)



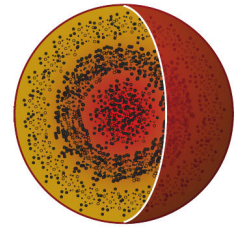
**Planetary  
atom model**

Rutherford (1911)



**Orbital  
atom model**

Bohr (1913)



**Quantum mechanical  
atom model**

Schrödinger (1926)

**Energy Encyclopedia**.com

## A revolutionary theory that escapes human intuition

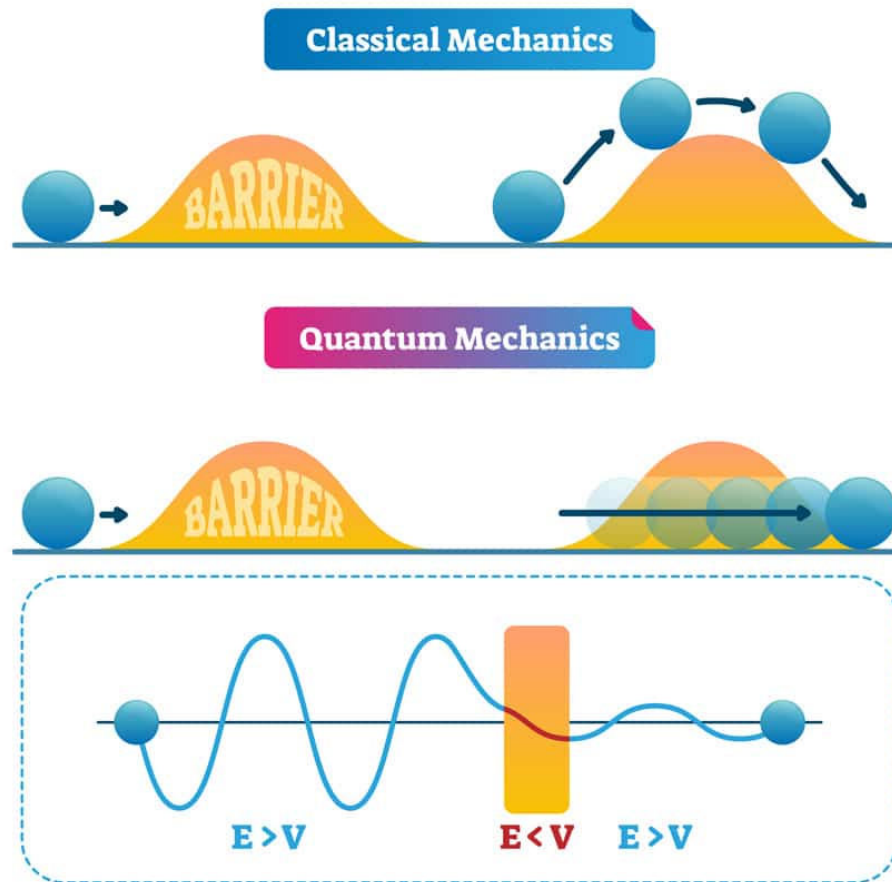


Image Courtesy: normaals/Getty Images

## Wave-particle duality

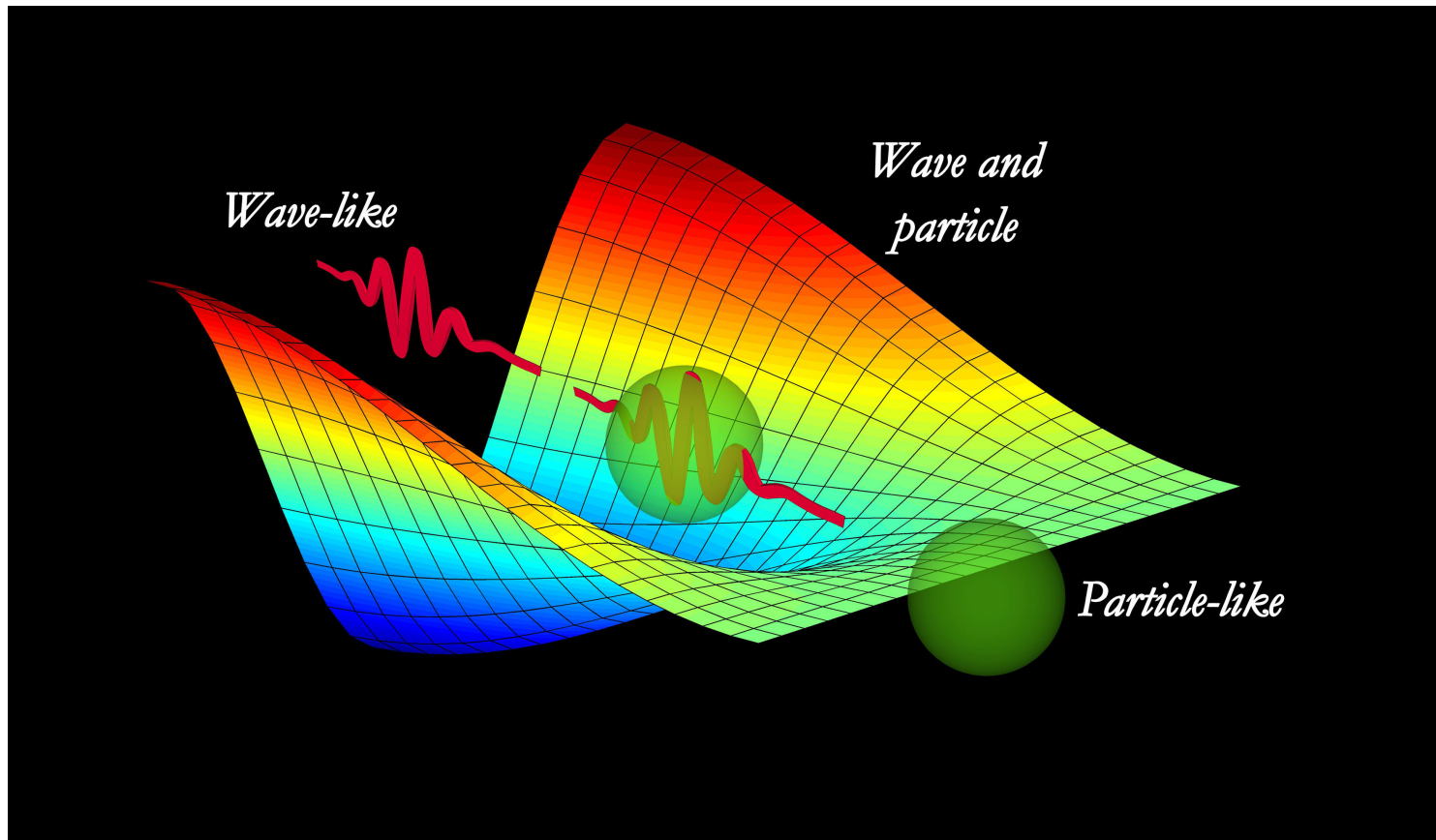


Image credit: S. Tanzilli, CNRS

## Wanted! But dead or alive?

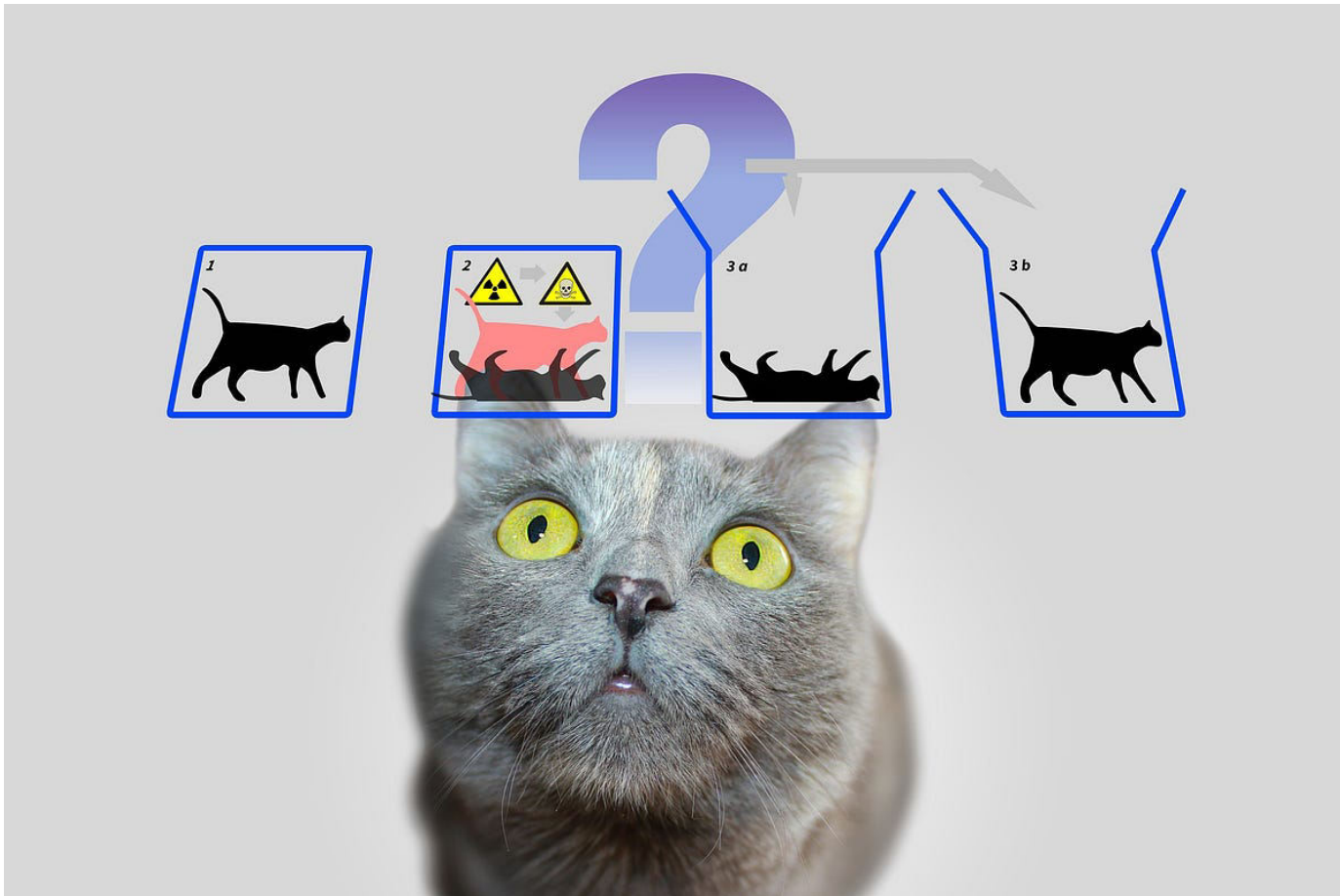
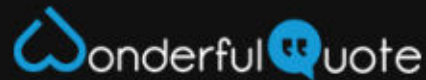


Image by [Gerd Altmann](#), via [Pixabay](#)

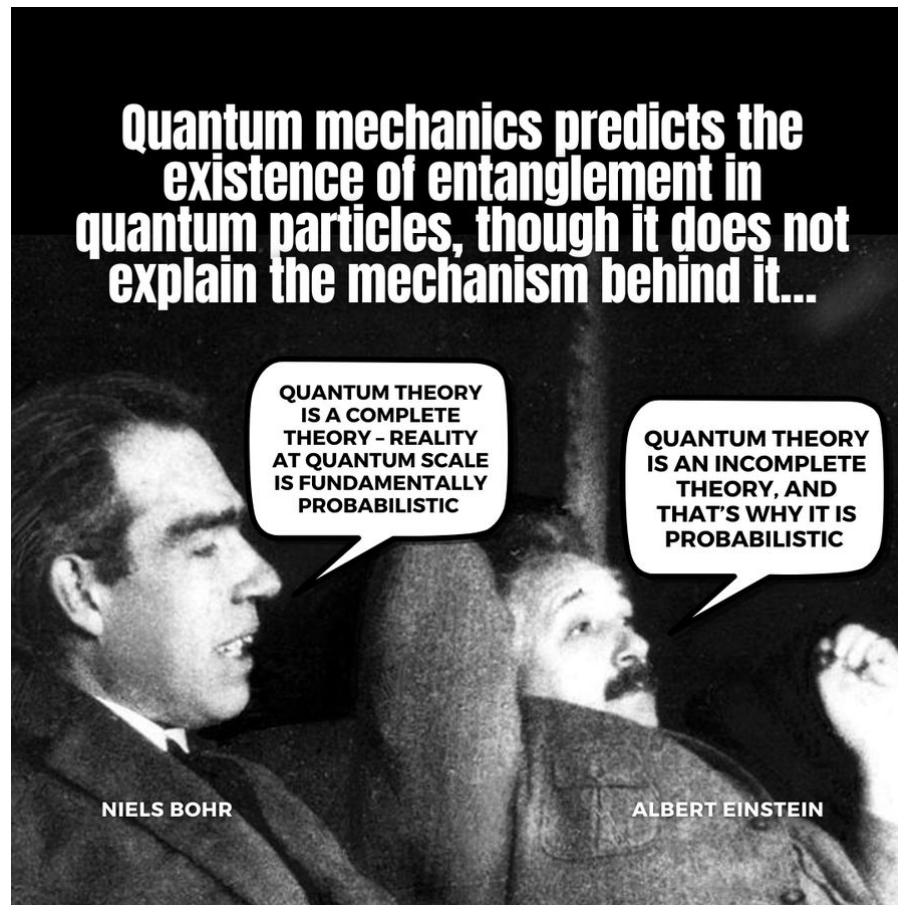
## The philosophical problems of a probabilistic theory

God does not play  
dice with the  
universe.

*Albert Einstein*

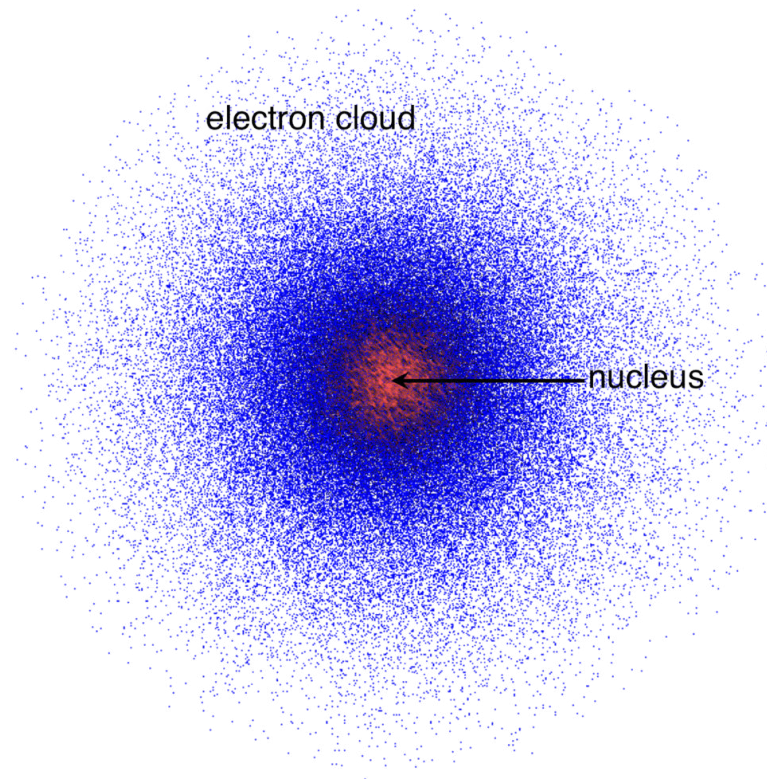


## A solution that seemed far from complete (and still does)



## The concept of electronic cloud

The electrons have a probability to be located in a given region around the nucleus



from [cosmosatyourdoorstep.com](http://cosmosatyourdoorstep.com)

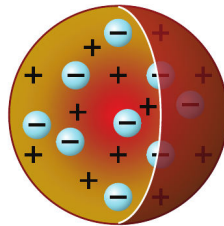
## The end of a very long journey

An atom can be modeled as a nucleus with a system of interacting electrons



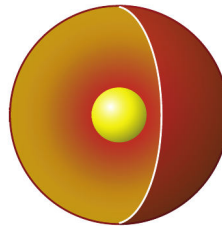
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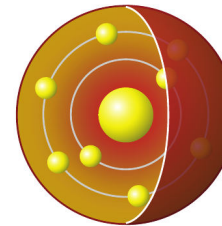
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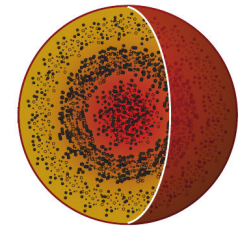
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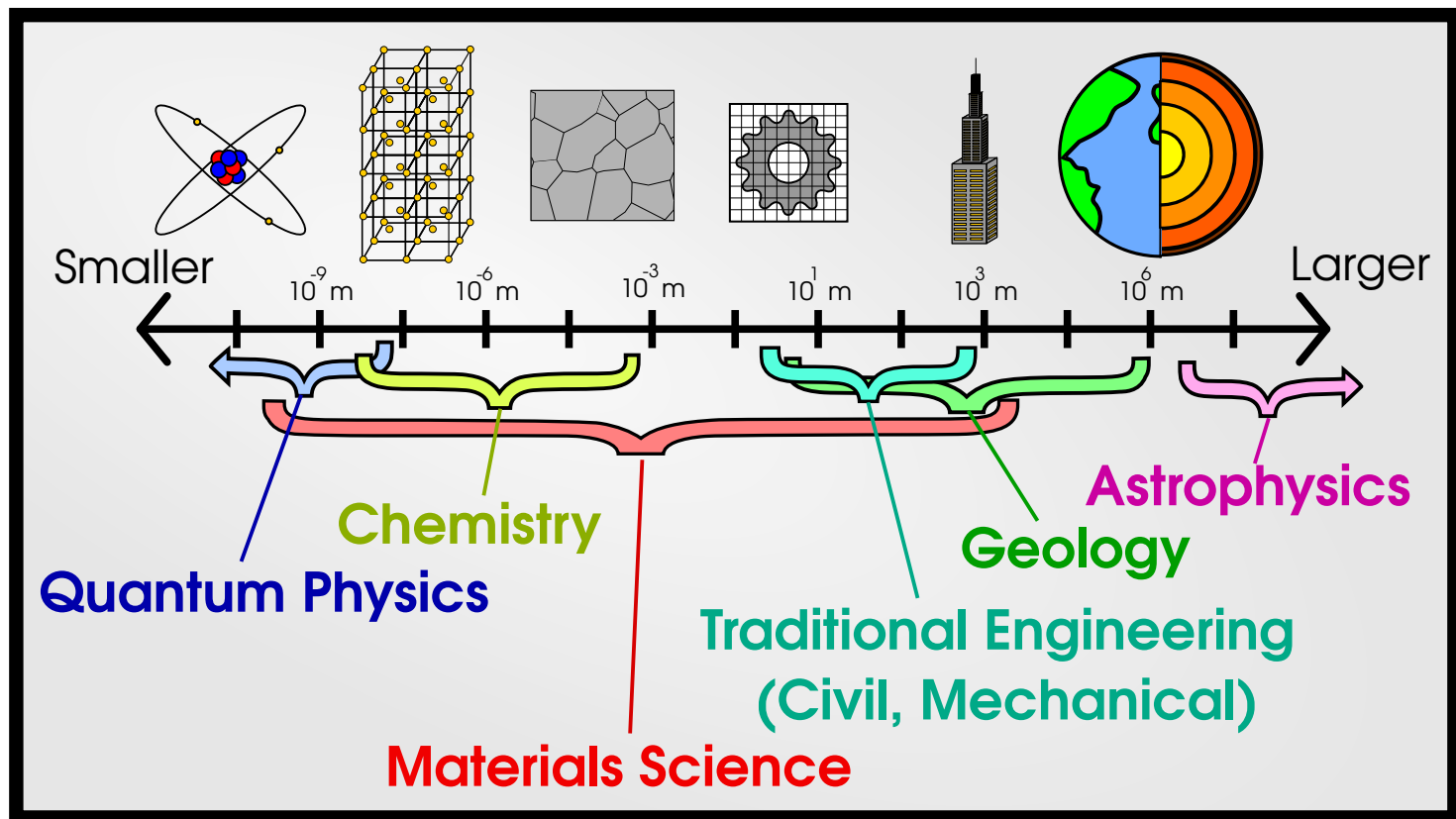
[Energy Encyclopedia.com](https://www.energyencyclopedia.com)

An atom is not the final indivisible component but remains still important

Toward multi-scale physics

Can we go from small to big length scales?

What if had infinite mathematical or computational power?

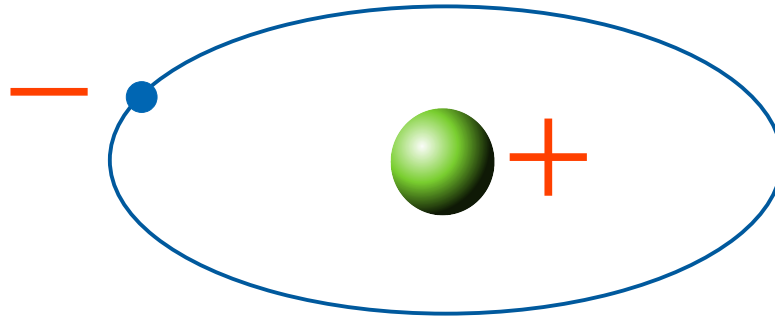


from mstudent.com

A bit of mathematics...

## The rules of the game: the atomic model

An atom can be modeled as a nucleus with a system of interacting electrons



We must “simply” solve the Schrodinger equation of electrons and nucleus



are much bigger than



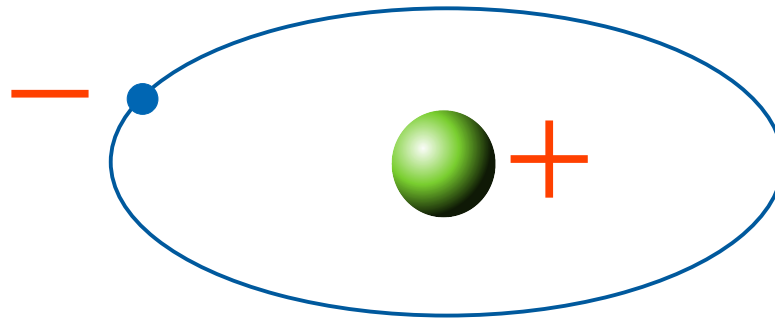
and also heavier!

As in classical mechanics, the electrons can be decoupled

A bit of mathematics...

## The rules of the game: the atomic model

An atom can be modeled as a nucleus with a system of interacting electrons



We must “simply” solve the Schrodinger equation of electrons and nucleus

Decoupling the nucleus, we are left with the equation for electrons only:

$$\left[ T_e + V_{ne} + V_{ee} \right] \psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) = E \psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$$

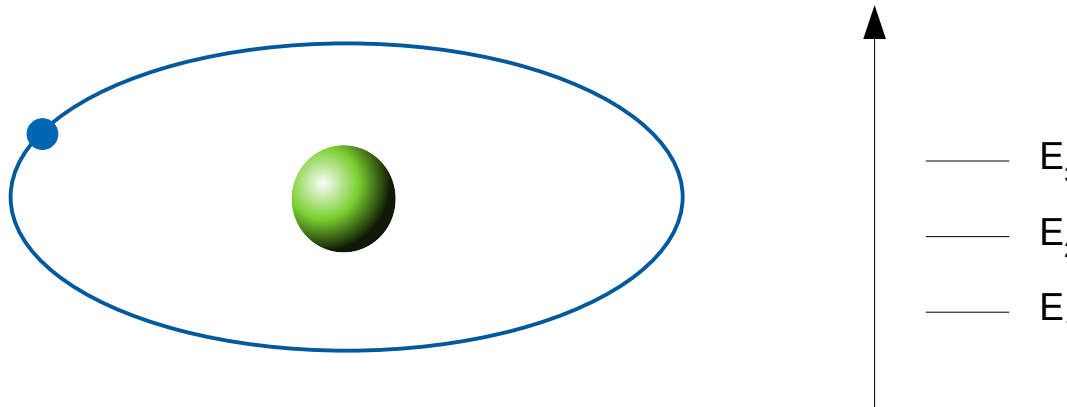
The crucial part is the electronic wavefunction  $\psi$

A bit of mathematics...

## The rules of the game: the atomic model

An atom can be modeled as a nucleus with a system of interacting electrons

The Schrodinger equation for the electrons is exactly or numerically solvable

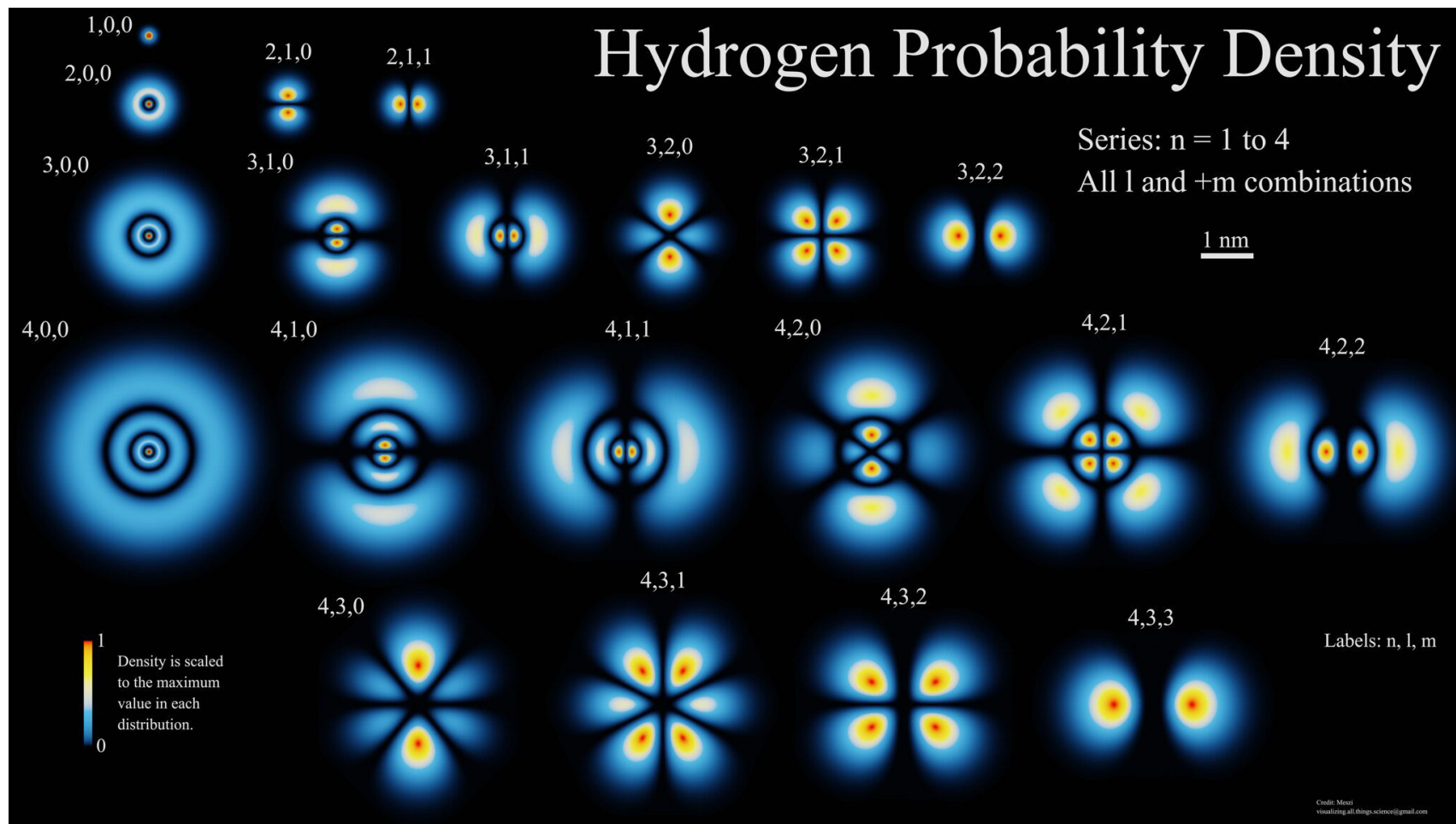


The electrons can occupy a set of discrete energy levels (**bound states**)

Electronic shells: 1s 2s 2p 3s 3p 3d 4s 4p 4d 4f 5s 5p 5d 5f ...

A bit of mathematics...

**The only available analytical solution!**





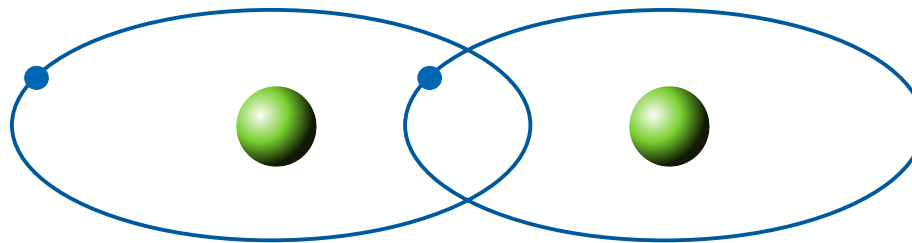
## The rules of the game: from atoms to solids

What does it happen when we bring a few atoms together?



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What does it happen when we bring a few atoms together?



A chemical bond is formed and keeps these ions together

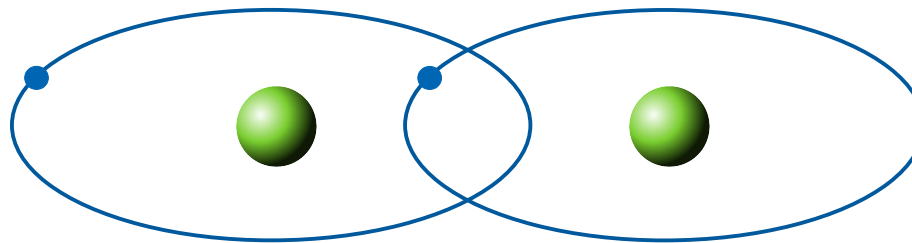
Small molecules or clusters are numerically solvable in quantum chemistry

- By decoupling electrons and nuclei, one obtains the electronic SE

$$\left[ T_e + V_{ne} + V_{ee} \right] \psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) = E \psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$$

## The rules of the game: from atoms to solids

What does it happen when we bring a few atoms together?



A chemical bond is formed and keeps these ions together

Small molecules or clusters are numerically solvable in quantum chemistry

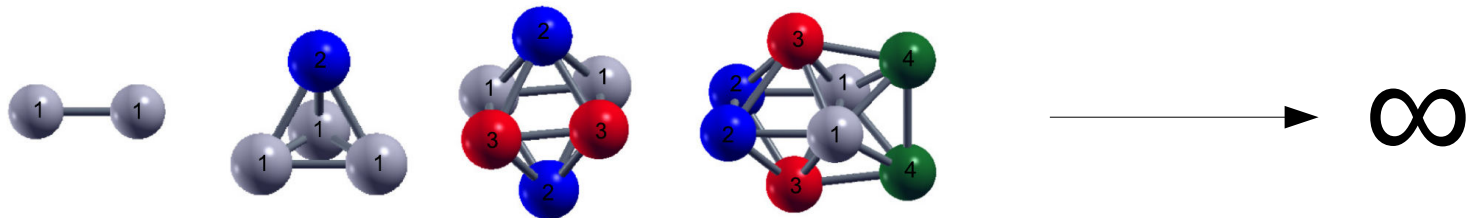
- By decoupling electrons and nuclei, one obtains the electronic SE
- Electronic problem solved via configuration interaction or coupled clusters
- The atomic positions are optimized

## The rules of the game: from atoms to solids

Atoms and molecules are still elementary components

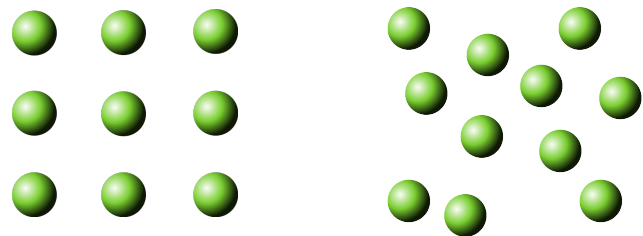
But how do we go from there to materials?

Let us consider a cluster of atoms, where we add one atom at the time



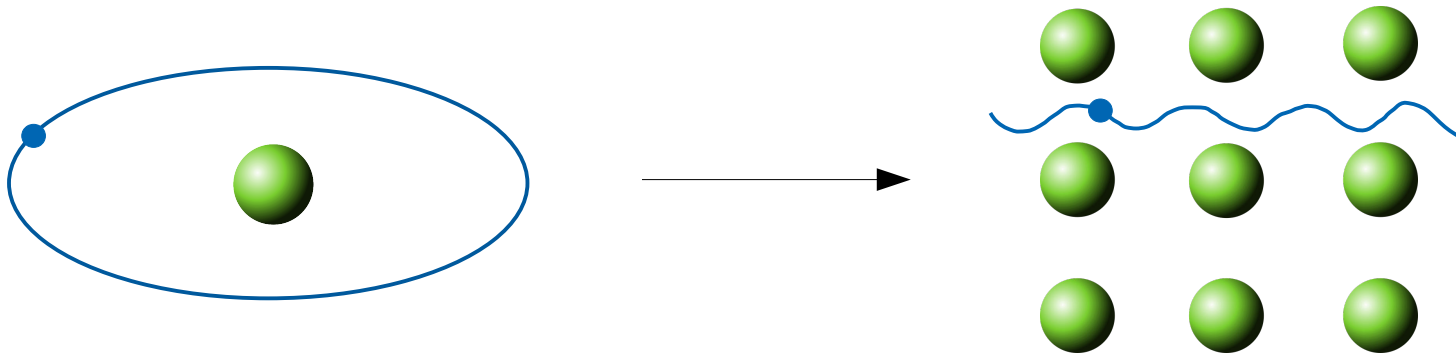
A periodic structure can be obtained

Crystalline vs amorphous solids



## The rules of the game: from atoms to solids

A crystalline solid is composed by many atoms in a regular arrangement



A few interacting electrons

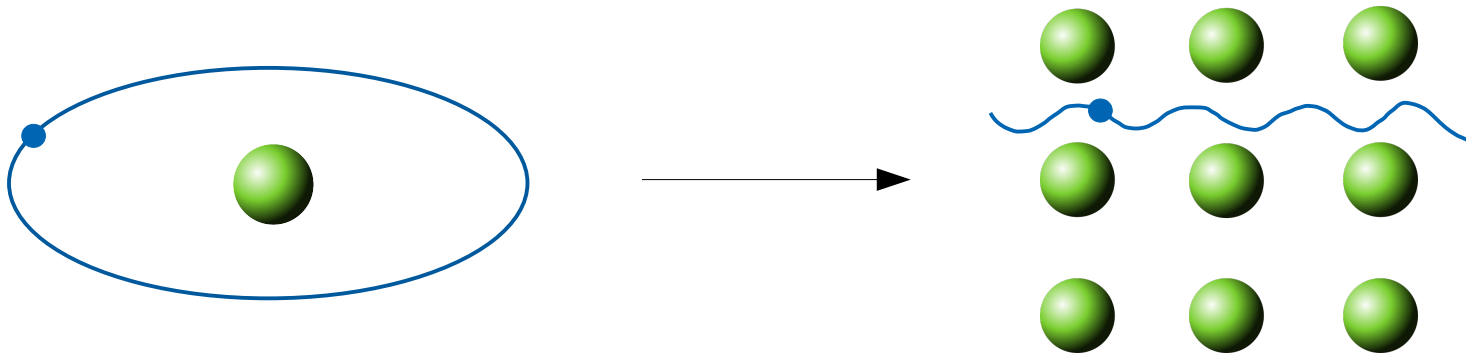
$10^{23}$  electrons

$10^{23}$  electrons = 1000000000000000000000000 electrons

Let us remember that we must solve for an equation for  $\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$

# The rules of the game: from atoms to solids

A crystalline solid is composed by many atoms in a regular arrangement



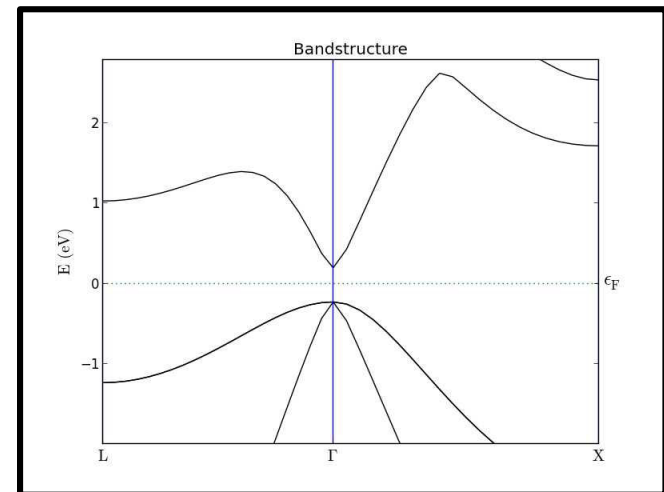
Too many particles ( $\sim 10^{23}$ ) for a solution!

In one-particle approximation, we obtain

1) delocalized electrons

2) energy bands

But how adequate is this simple picture?



## Solving the many-electron problem in solids

Density-functional theory (DFT) is the most used electronic structure theory

DFT is based on two fundamental concepts

1) focus is shifted from the wavefunction  $\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$  to the electron density  $n(\mathbf{r})$

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1000000000000000000000000000000 variables

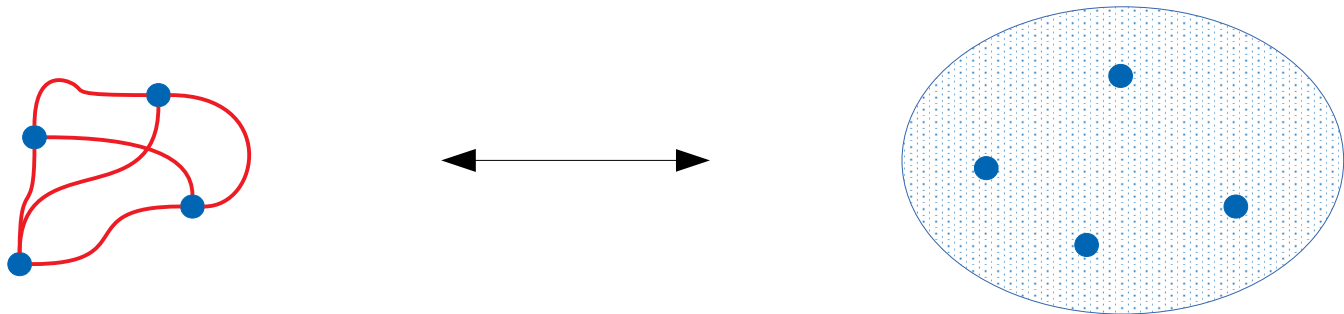
1 variable

## Solving the many-electron problem in solids

Density-functional theory (DFT) is the most used electronic structure theory

DFT is based on two fundamental concepts

- 1) focus is shifted from the wavefunction  $\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$  to the electron density  $n(\mathbf{r})$
- 2) a mapping onto a system of non interacting electrons in an effective potential



The key advantage is that we do not need to simplify the initial problem!

## Solving the many-electron problem in solids

Density-functional theory (DFT) is the most used electronic structure theory

It is a very general approach, from atoms, molecules and clusters to solids

Provides access to a variety of properties

- equilibrium crystal structure

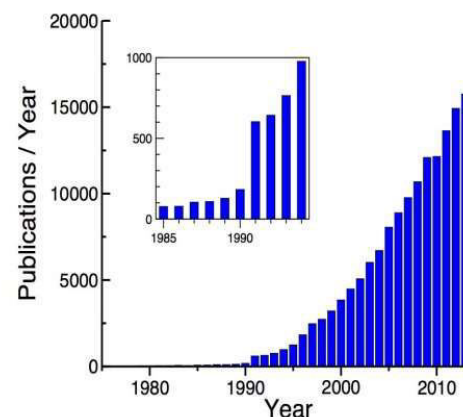
- elastic properties

- magnetic properties

- optical properties

- thermoelectric properties

- ...



A high accuracy and a high computational efficiency

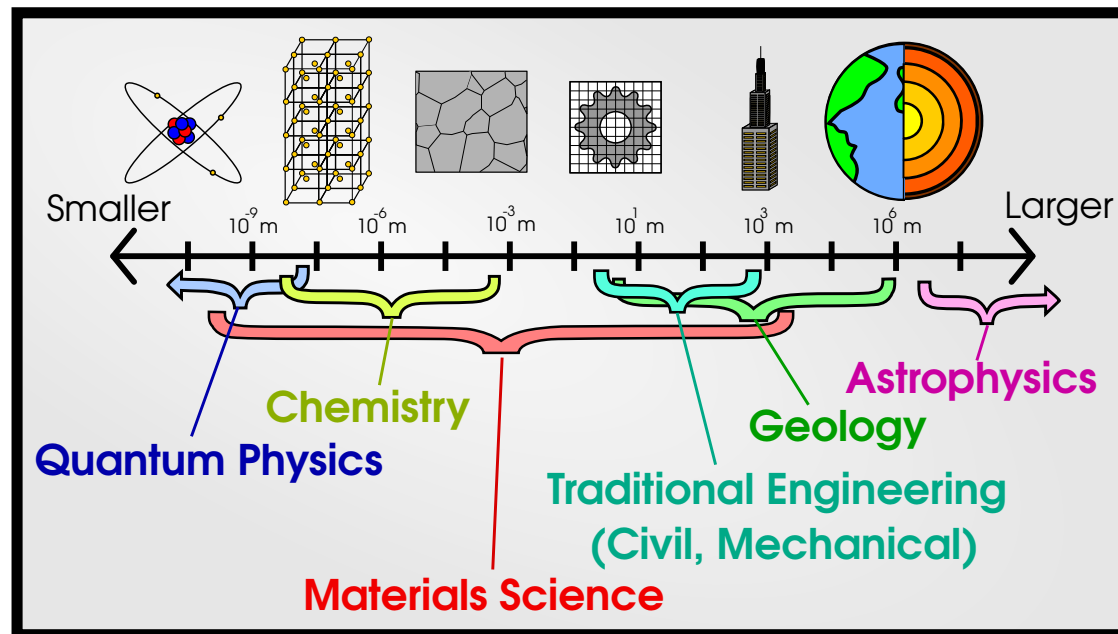
What is the limitation of not having the full electronic wavefunction?

## Solving the many-electron problem in solids

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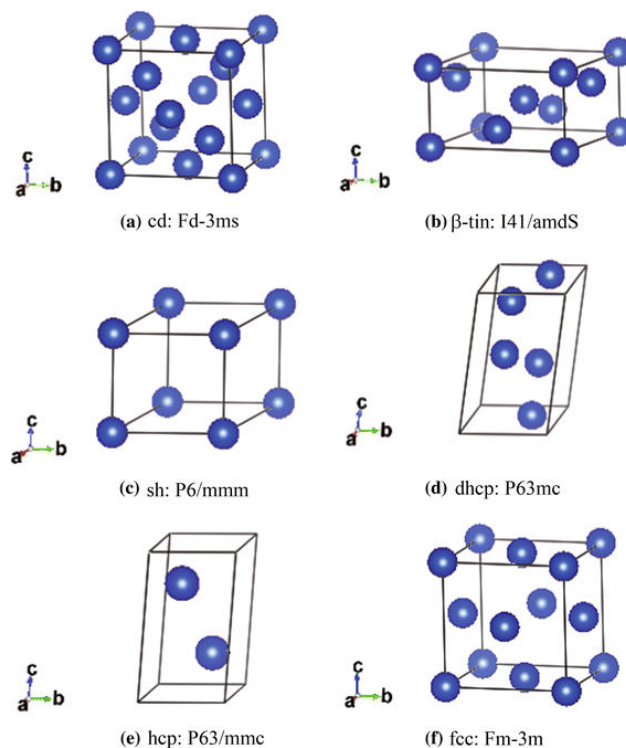
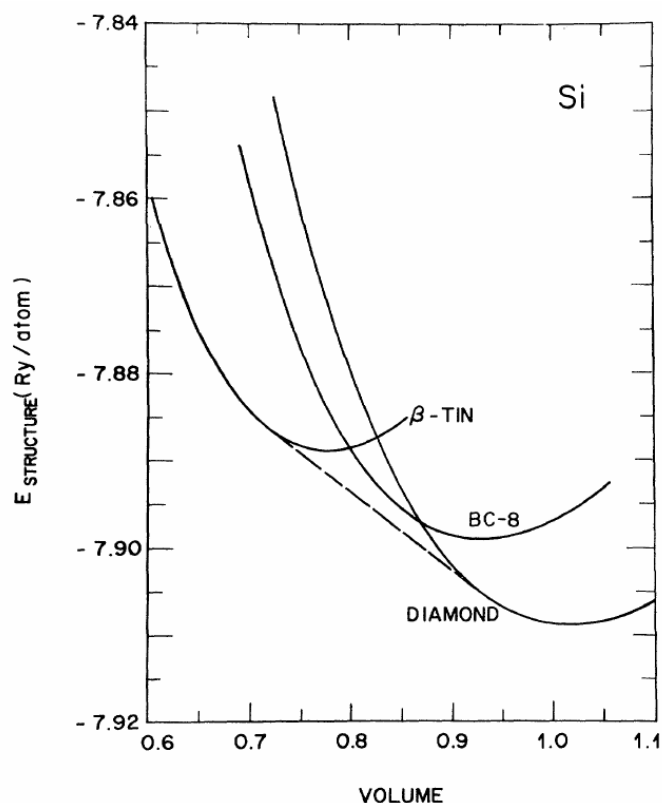
DFT provides a description of the macroscopic world in **ideal conditions**



from mstudent.com

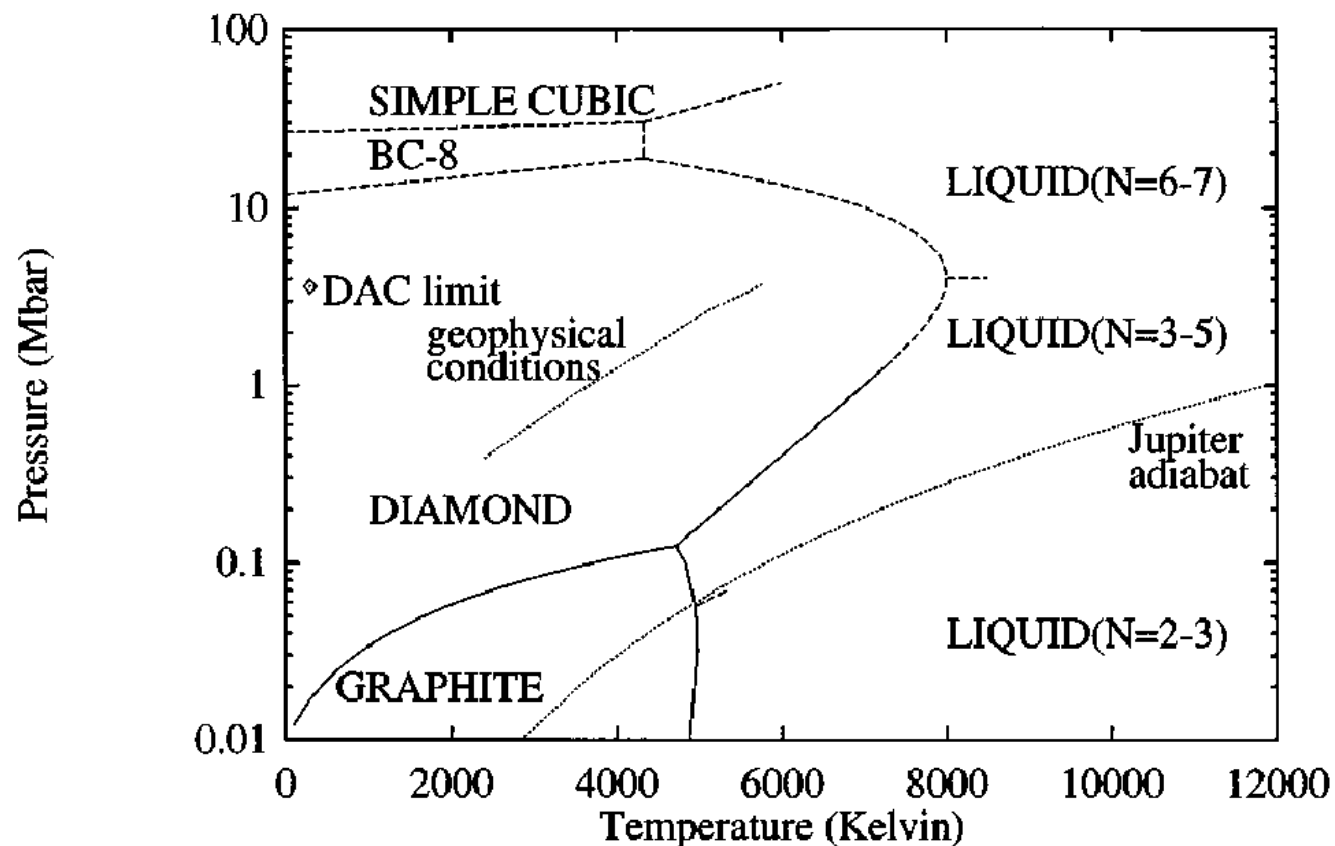
A few examples from modern research

## Equilibrium crystal structure of silicon under pressure



A few examples from modern research

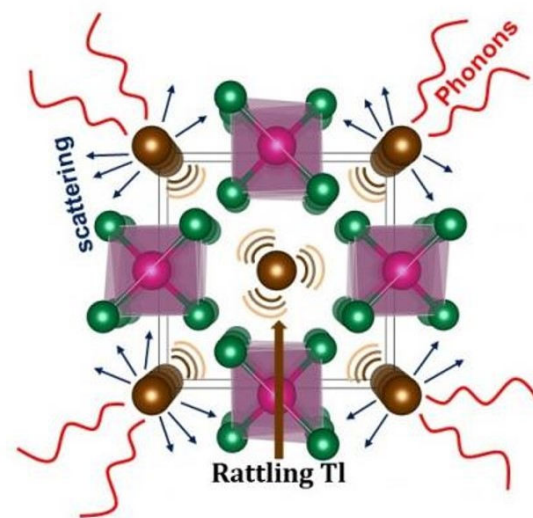
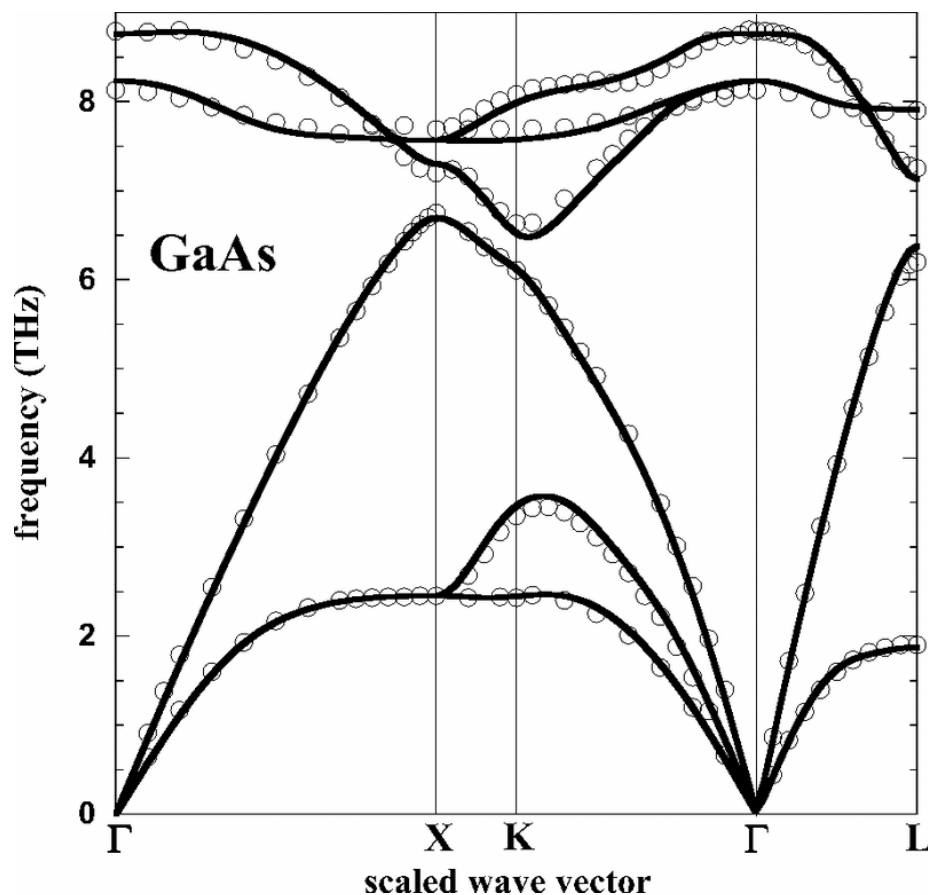
## Phase diagram of carbon



R. Martin, Electronic Structure, Cambridge University Press

A few examples from modern research

## Energy of the lattice vibrations in GaAs



Angew. Chem. Int. Ed. **60**, 4259 (2021)

A few examples from modern research

## Prediction of novel 2-dimensional materials

TABLE II. List of two-dimensional dichalcogenides found by our search algorithm with the ICSD number of the corresponding bulk material and with our calculated band gap (in eV). The corresponding magnetic state (AFM refers to an antiferromagnetic ordering) is given in the last column to the right. (The cells of the table are left blank if the material is not magnetically ordered.)

2D chemical formula	3D ICSD number	Gap (eV)	Magnetism	2D chemical formula	3D ICSD number	Gap (eV)	Magnetism
YS <sub>2</sub>	651404	Metal		CrTe <sub>2</sub>	152836	Metal	FM
TiS <sub>2</sub>	651178	0.02		MoS <sub>2</sub>	644245	1.6	
TiSe <sub>2</sub>	173923	Metal		MoSe <sub>2</sub>	644334	1.4	
TiTe <sub>2</sub>	653071	Metal		MoTe <sub>2</sub>	015431	1.15	
ZrS <sub>2</sub>	651465	1.1		WS <sub>2</sub>	202366	1.8	
ZrSe <sub>2</sub>	652236	0.4		WSe <sub>2</sub>	040752	1.5	
ZrTe <sub>2</sub>	653213	Metal		WTe <sub>2</sub>	073323	Metal	
HfS <sub>2</sub>	638847	1.3		TcS <sub>2</sub>	081816	1.2	
HfSe <sub>2</sub>	638899	0.6		ReS <sub>2</sub>	075459	1.4	
HfTe <sub>2</sub>	638959	Metal		ReSe <sub>2</sub>	081813	1.3	
VS <sub>2</sub>	651361	Metal	FM	CoTe <sub>2</sub>	625401	Metal	
VSe <sub>2</sub>	652158	Metal	FM	RhTe <sub>2</sub>	650448	Metal	
VT <sub>2</sub>	603582	Metal	FM	IrTe <sub>2</sub>	033934	Metal	
NbS <sub>2</sub>	645307	Metal		NiTe <sub>2</sub>	159382	Metal	
NbSe <sub>2</sub>	645369	Metal		PdS <sub>2</sub>	166276	1.1	
NbTe <sub>2</sub>	645529	Metal		PdSe <sub>2</sub>	170327	1.3	
TaS <sub>2</sub> -AB	651089	Metal		PdTe <sub>2</sub>	649016	0.2	
TaS <sub>2</sub> -AA	651092	Metal		PtS <sub>2</sub>	649534	1.8	
TaSe <sub>2</sub> -AB	651948	Metal		PtSe <sub>2</sub>	649589	1.4	
TaSe <sub>2</sub> -AA	651950	Metal		PtTe <sub>2</sub>	649747	0.8	
TaTe <sub>2</sub>	014390	Metal		SiTe <sub>2</sub>	652385	Metal	
CrS <sub>2</sub>	075420	Metal	AFM	SnS <sub>2</sub>	650992	1.6	
CrSe <sub>2</sub>	626718	Metal	FM	SnSe <sub>2</sub>	651910	0.8	

Phys. Rev. X 3, 031002 (2013)

## A few examples from modern research

# Experimental confirmation after a decade

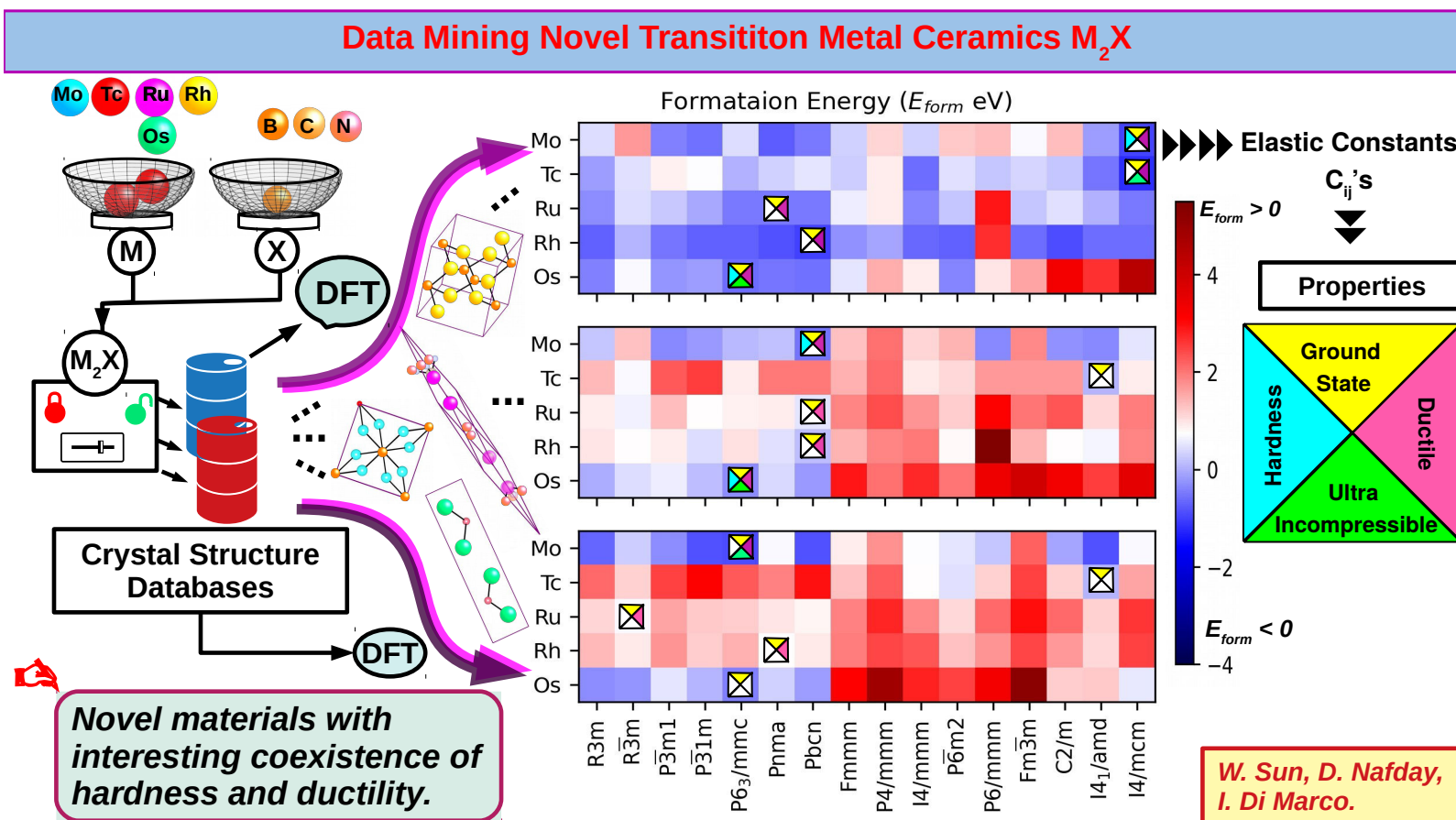
**Table 1** Theoretical and experimental results of magnetic ground states (MGS), MAE and critical temperature  $T_c$  for various 2D magnetic crystals

Sample	MGS	MAE [ $\mu\text{eV}$ ]	$T_c$ [K]	Sample	MGS	MAE [ $\mu\text{eV}$ ]	$T_c$ [K]
<b>CrI<sub>3</sub></b> <sup>19,38</sup>	FM	803.6 (Is)	46, 45 <sup>11</sup>	<b>VS<sub>2</sub></b> <sup>92</sup>	FM	~310 (XY)	90
<b>CrBr<sub>3</sub></b> <sup>38,99</sup>	FM	159.5 (Is)	41, 33 <sup>tf</sup>	<b>VSe<sub>2</sub></b> <sup>21</sup>	FM	— (XY)	>300 <sup>11</sup>
<b>CrCl<sub>3</sub></b> <sup>26,99</sup>	FM	— (XY)	16.8, 15 <sup>tf</sup>	<b>MnS<sub>2</sub></b> <sup>100</sup>	FM	—	225
<b>VCl<sub>3</sub></b> <sup>101</sup>	FM	—	80	<b>MnSe<sub>2</sub></b> <sup>22,100</sup>	FM	—	250, ~300 <sup>tf</sup>
<b>VI<sub>3</sub></b> <sup>101,102</sup>	FM	—	98, 50 <sup>tf</sup>	<b>VCl<sub>2</sub></b> <sup>103</sup>	AFM	40	—
<b>VBr<sub>3</sub></b> <sup>48,104</sup>	FM	450 (Is)	190, 27 <sup>tf</sup>	<b>VI<sub>2</sub></b> <sup>103</sup>	AFM	60	—
<b>NiBr<sub>3</sub></b> <sup>48</sup>	FM	377 (Is)	100	<b>VBr<sub>2</sub></b> <sup>103</sup>	AFM	620	—
<b>PdBr<sub>3</sub></b> <sup>48</sup>	FM	659 (Is)	110	<b>FeCl<sub>2</sub></b> <sup>89</sup>	FM	~60 (XY)	165
<b>FeBr<sub>3</sub></b> <sup>48</sup>	nAFM	—	70	<b>FeBr<sub>2</sub></b> <sup>89</sup>	FM	— (XY)	210
<b>Cr<sub>2</sub>Ge<sub>2</sub>Te<sub>6</sub></b> <sup>18</sup>	FM	<1 (H)	28 <sup>t2</sup>	<b>FeI<sub>2</sub></b> <sup>89</sup>	FM	— (XY)	122
<b>Cr<sub>2</sub>Si<sub>2</sub>Te<sub>6</sub></b> <sup>50,105</sup>	FM	419 (Is)	90, 31 <sup>tf</sup>	<b>NiI<sub>2</sub></b> <sup>25,106</sup>	FM	1.12 (Is)	86, 35–55 <sup>tf</sup>
<b>Cr<sub>2</sub>Sn<sub>2</sub>Te<sub>6</sub></b> <sup>50</sup>	FM	69 (Is)	170	<b>CoCl<sub>2</sub></b> <sup>25</sup>	FM	1.85 (Is)	25
<b>Fe<sub>3</sub>GeTe<sub>2</sub></b> <sup>20</sup>	FM	2000 (Is)	68 <sup>t1</sup>	<b>MnCl<sub>2</sub></b> <sup>25</sup>	AFM	111 (Is)	33
<b>VPS<sub>3</sub></b> <sup>53</sup>	nAFM	— (Is)	570	<b>MnBr<sub>2</sub></b> <sup>25</sup>	AFM	322 (Is)	40
<b>CrPS<sub>3</sub></b> <sup>53</sup>	FM	~160 (Is)	130	<b>Fe<sub>2</sub>C</b> <sup>65</sup>	FM	22.8 (XY)	861
<b>MnPS<sub>3</sub></b> <sup>53</sup>	nAFM	~1000 (Is)	200, 79 <sup>tf</sup>	<b>MnNO<sub>2</sub></b> <sup>69</sup>	FM	63 (Is)	67
<b>FePS<sub>3</sub></b> <sup>53,169</sup>	zAFM	— (Is)	260, 118 <sup>t1</sup>	<b>CrNO<sub>2</sub></b> <sup>69</sup>	FM	22 (Is)	53
<b>CoPS<sub>3</sub></b> <sup>53,107</sup>	zAFM	>1000 (Is)	330	<b>MnNF<sub>2</sub></b> <sup>69</sup>	FM	2 (XY)	1128
<b>NiPS<sub>3</sub></b> <sup>53,108</sup>	zAFM	— (Is)	560, 130 <sup>t2</sup>	<b>Ti<sub>2</sub>NO<sub>2</sub></b> <sup>69</sup>	FM	0.78 (H)	—
<b>CrCuP<sub>2</sub>S<sub>6</sub></b> <sup>58,59</sup>	FM	28 (XY)	64 <sup>tf</sup>	<b>Hf<sub>2</sub>VC<sub>2</sub>F<sub>2</sub></b> <sup>70</sup>	AFM	—	313
<b>CrCuP<sub>2</sub>Se<sub>6</sub></b> <sup>58</sup>	FM	242 (Is)	—	<b>MnBi<sub>2</sub>Te<sub>4</sub></b> <sup>109</sup>	AFM	— (Is)	21 <sup>t7</sup>
<b>CrOCl</b> <sup>83</sup>	FM	~110 (Is)	160	<b>CoGa<sub>2</sub>S<sub>4</sub></b> <sup>75</sup>	FM	47 (XY)	195
<b>CrOBr</b> <sup>83</sup>	FM	~290 (Is)	129	<b>CoGa<sub>2</sub>Se<sub>4</sub></b> <sup>75</sup>	FM	84 (XY)	516
<b>CrScl</b> <sup>84</sup>	FM	20 (Is)	108	<b>CoGa<sub>2</sub>Te<sub>4</sub></b> <sup>75</sup>	FM	634 (XY)	570
<b>CrSBr</b> <sup>84</sup>	FM	20 (Is-x)	127	<b>FeTe</b> <sup>82</sup>	AFM	—	45–70 <sup>tf</sup>
<b>CrSI</b> <sup>84</sup>	FM	60 (Is-y)	146	<b>MnNCl</b> <sup>84</sup>	FM	70 (Is)	238
<b>CrSeCl</b> <sup>84</sup>	FM	10 (Is-y)	118	<b>MnNBr</b> <sup>84</sup>	FM	20 (Is)	261
<b>CrSeBr</b> <sup>84</sup>	FM	4 (Is-y)	135	<b>MnNI</b> <sup>84</sup>	FM	930 (Is-x)	492
<b>CrSeI</b> <sup>84</sup>	FM	20 (Is-y)	164	<b>CrTeBr</b> <sup>84</sup>	FM	750 (Is)	187
<b>CrTeCl</b> <sup>84</sup>	FM	150 (Is-x)	248	<b>CrTeI</b> <sup>84</sup>	FM	550 (Is)	139

The synthesized 2D magnets are marked with bold sample names. In the  $T_c$  column, numbers without superscripts are theoretical results for monolayer samples. Superscripts 'tf' and 'tn' denote results of thin flakes and  $n$ -layer ( $n = 1, 2, 3, \dots$ ) experimental samples, respectively. Is, XY and H in the MAE column represent Ising, XY, and Heisenberg magnets, respectively, and Is-x and Is-y refer to the easy axis lying in the  $x$  or  $y$  directions in the plane of 2D anisotropic MXY magnets.

A few examples from modern research

## Big data studies and perspective for machine learning





A few examples from modern research

## **Applications for renewable energy**

### Photovoltaic Materials

→ to improve efficiency in energy conversion and stability to degradation

### Thermoelectric Materials

→ to predict novel materials and optimize for carrier concentration

### Hydrogen Storage Materials

→ to identify and optimize materials with better hydrogen interaction

### Catalysts for Renewable Energy

→ to design catalysts for processes like water splitting and CO<sub>2</sub> reduction

### Battery Materials

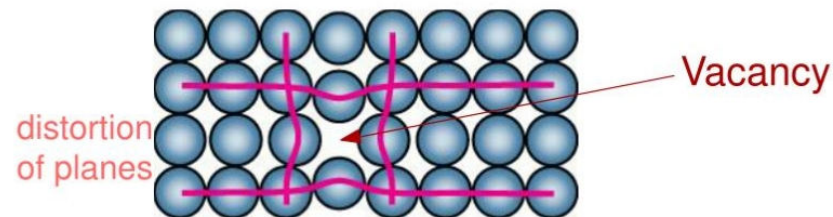
→ to discover materials with a higher energy density and a longer lifespan

## The role of defects and other imperfections

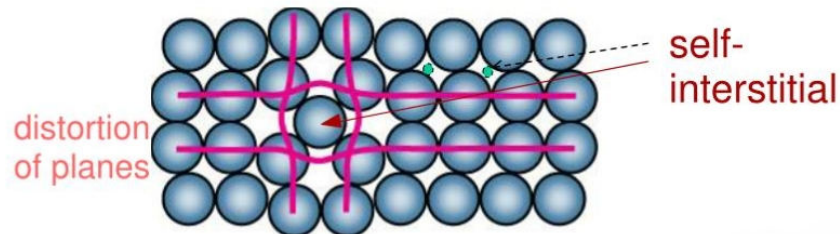
Perfect crystals do not represent reality very well...

### Point Defects

- **Vacancies:**  
-vacant atomic sites in a structure.



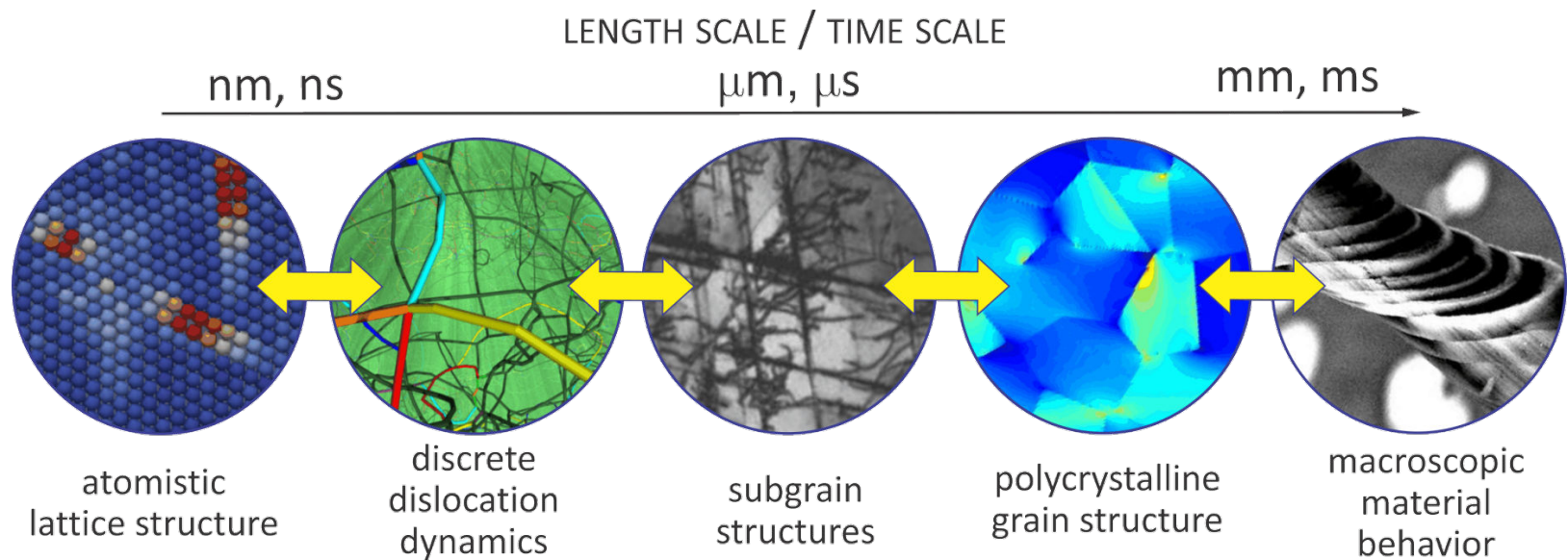
- **Self-Interstitials:**  
-"extra" atoms positioned between atomic sites.



from docs.neu.edu.tr

Matching macroscopic and microscopic systems

## Multi scale simulations are the next big step?



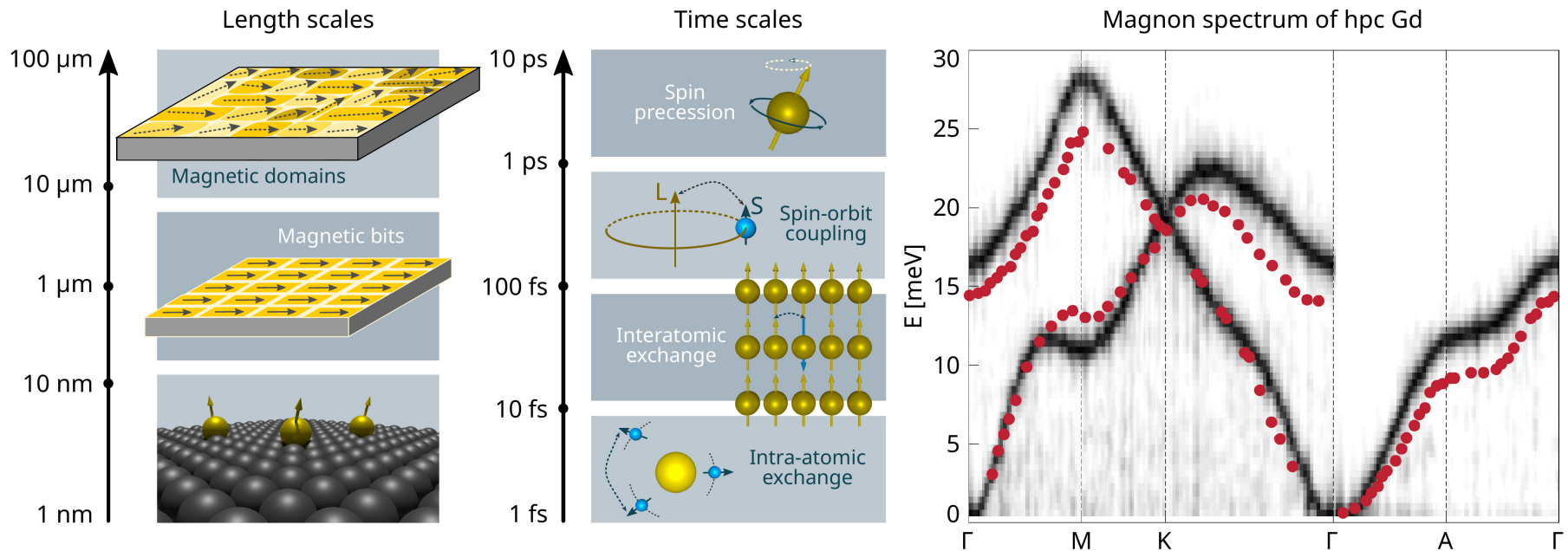
Computer-Aided Design **77**, 73 (2016)

A few examples from modern research

## Multi scale simulations are the next big step?

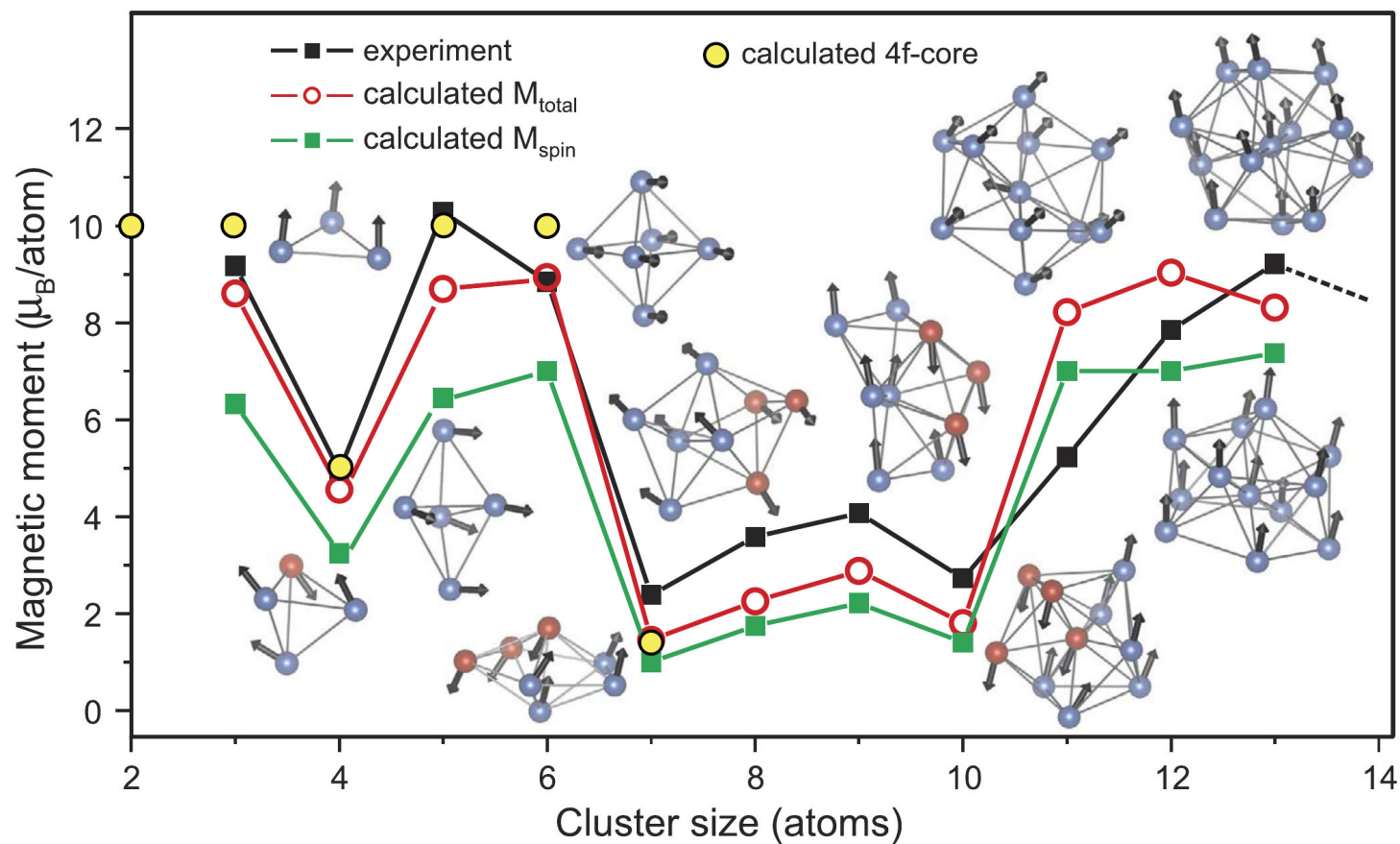
Multi-scale approaches are the frontier in magnetic simulations

These problems have a very high relevance for industrial applications



A few examples from modern research

## Magnetism of small Tb clusters



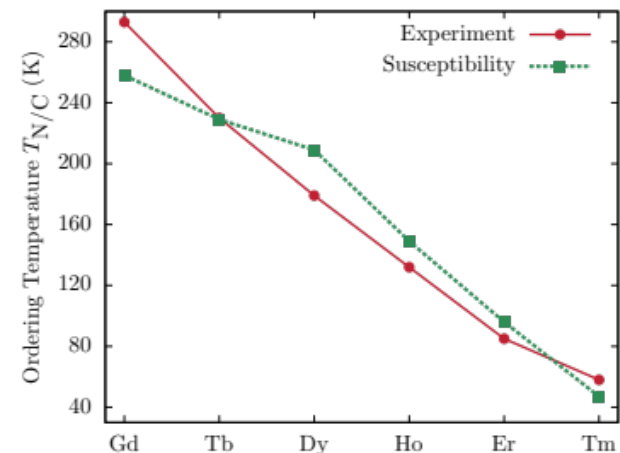
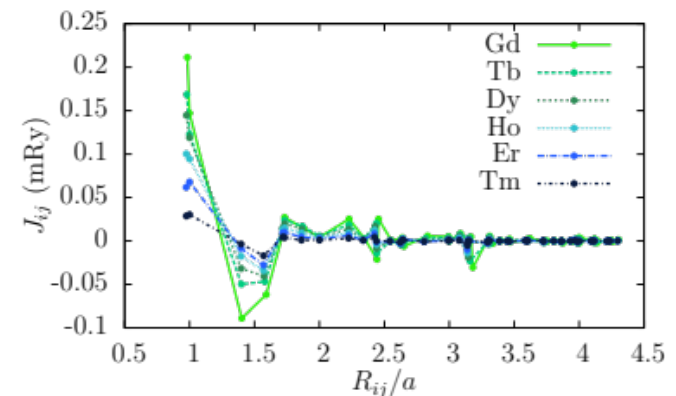
Scientific Reports **6**, 19676 (2016)

A few examples from modern research

## Magnetism in rare-earth elements

Multi-scale approaches are the frontier in magnetic simulations

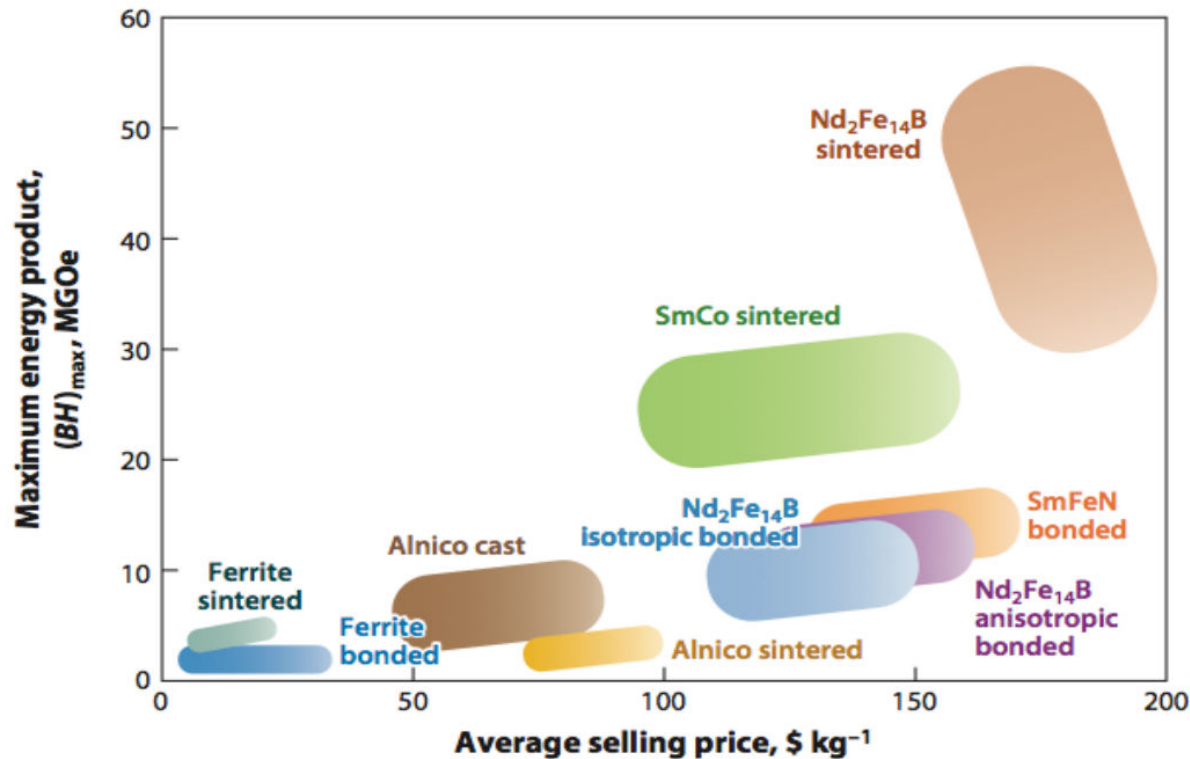
- Atomic-scale description in DFT
- Effective magnetic coupling
- Coupling used for spin dynamics
- Domain walls in micromagnetics



A few examples from modern research

## Magnetism in rare-earth elements

Multi-scale approaches are the frontier in magnetic simulations



from businessinsider.com.pl



from www.sw.siemens.com

## Solving the many-electron problem in solids

Density-functional theory (DFT) is the most used electronic structure theory

It is a very general approach, from atoms, molecules and clusters to solids

Provides access to a variety of properties

- equilibrium crystal structure

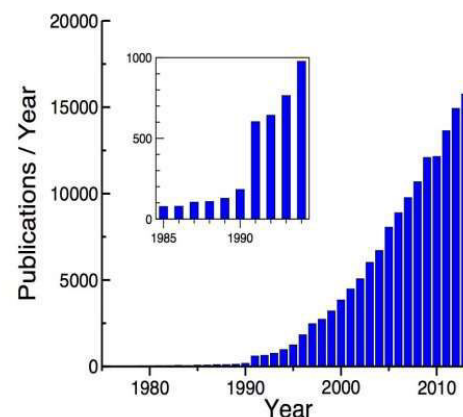
- elastic properties

- magnetic properties

- optical properties

- thermoelectric properties

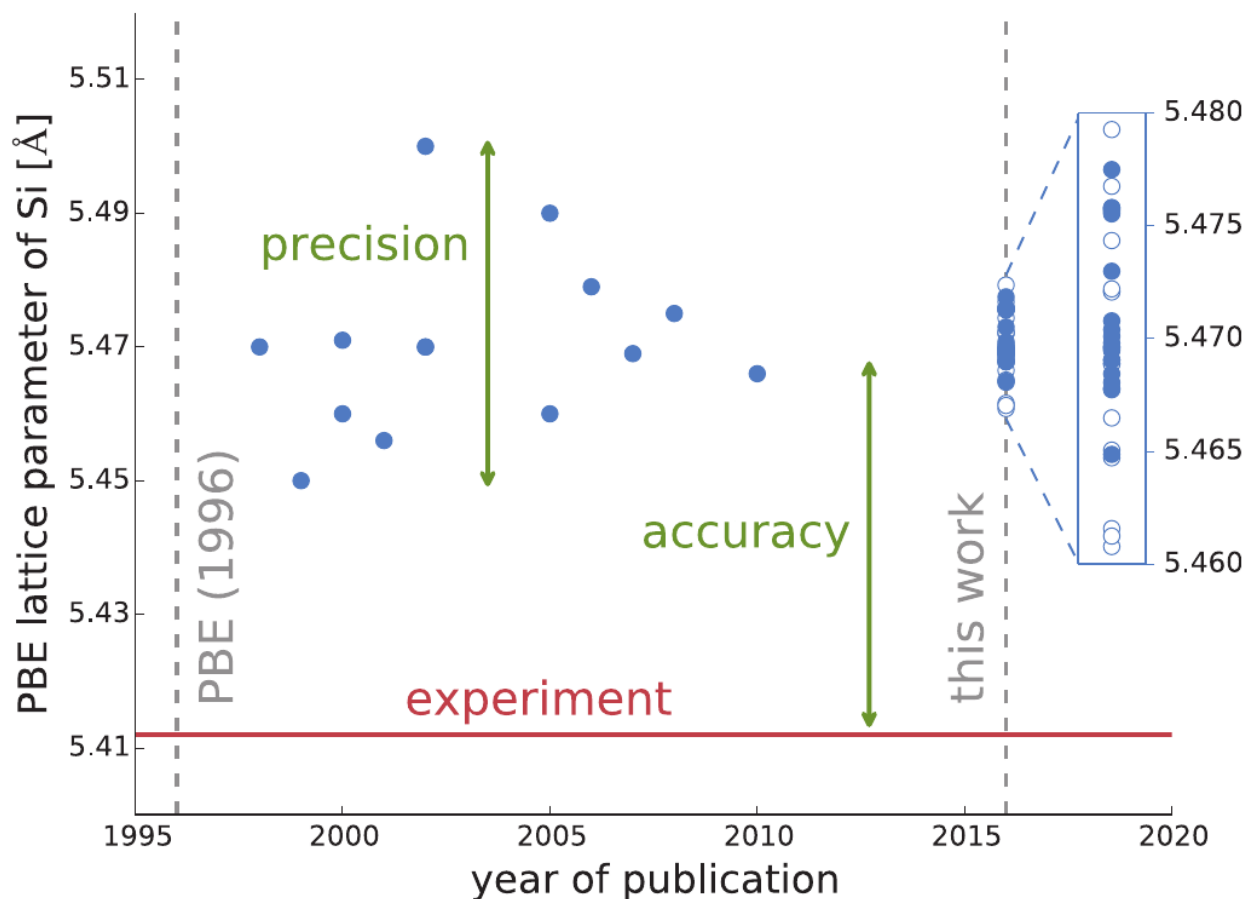
- ...



A high accuracy and a high computational efficiency

**But what are exactly the limitations of modeling based on DFT?**

## Lattice parameter of silicon and DFT accuracy

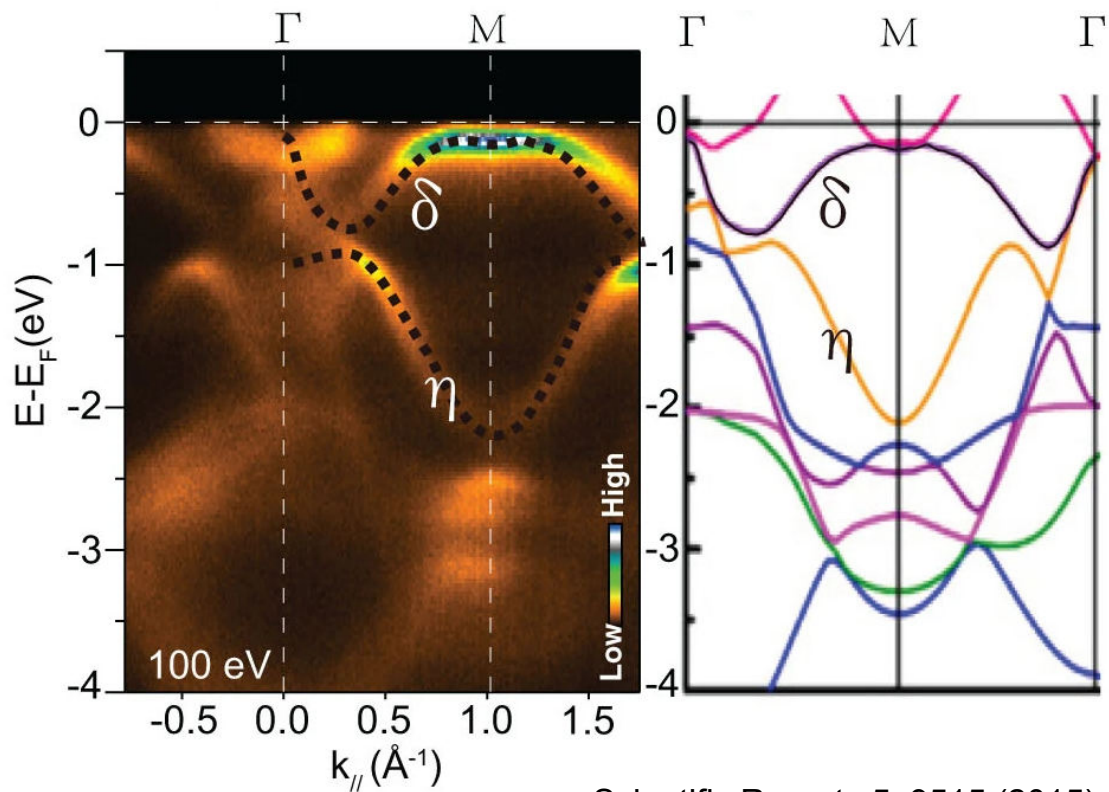


Science **351**, 6280 (2016)

## Excited-state properties

How does DFT results compare with ARPES?

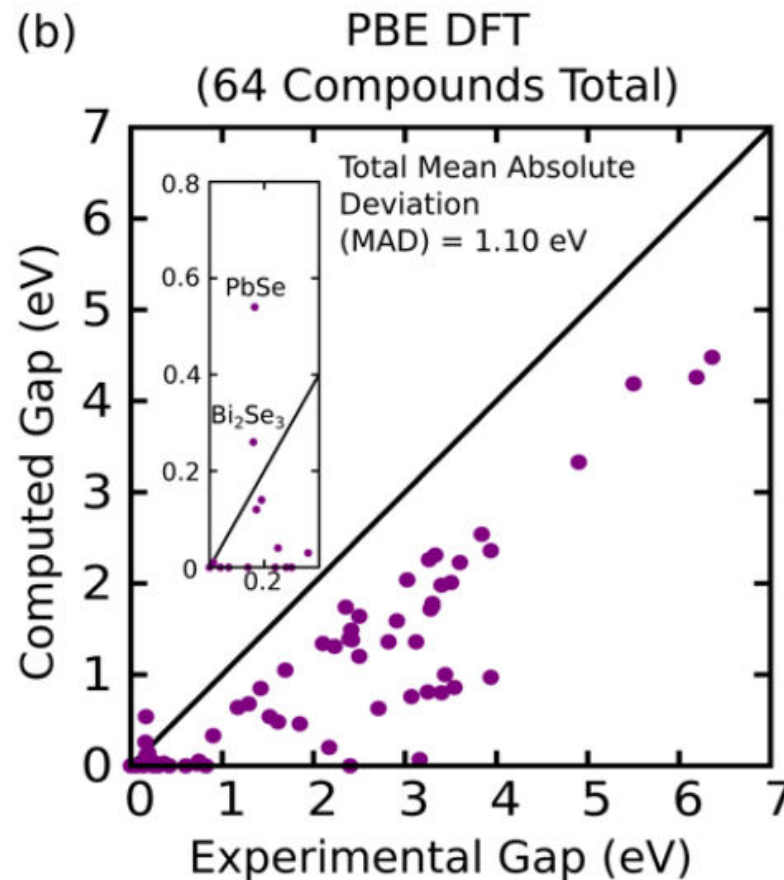
Let us see the charge-density wave state in  $\text{Na}_2\text{Ti}_2\text{Sb}_2\text{O}$



Scientific Reports **5**, 9515 (2015)

## Excited-state properties

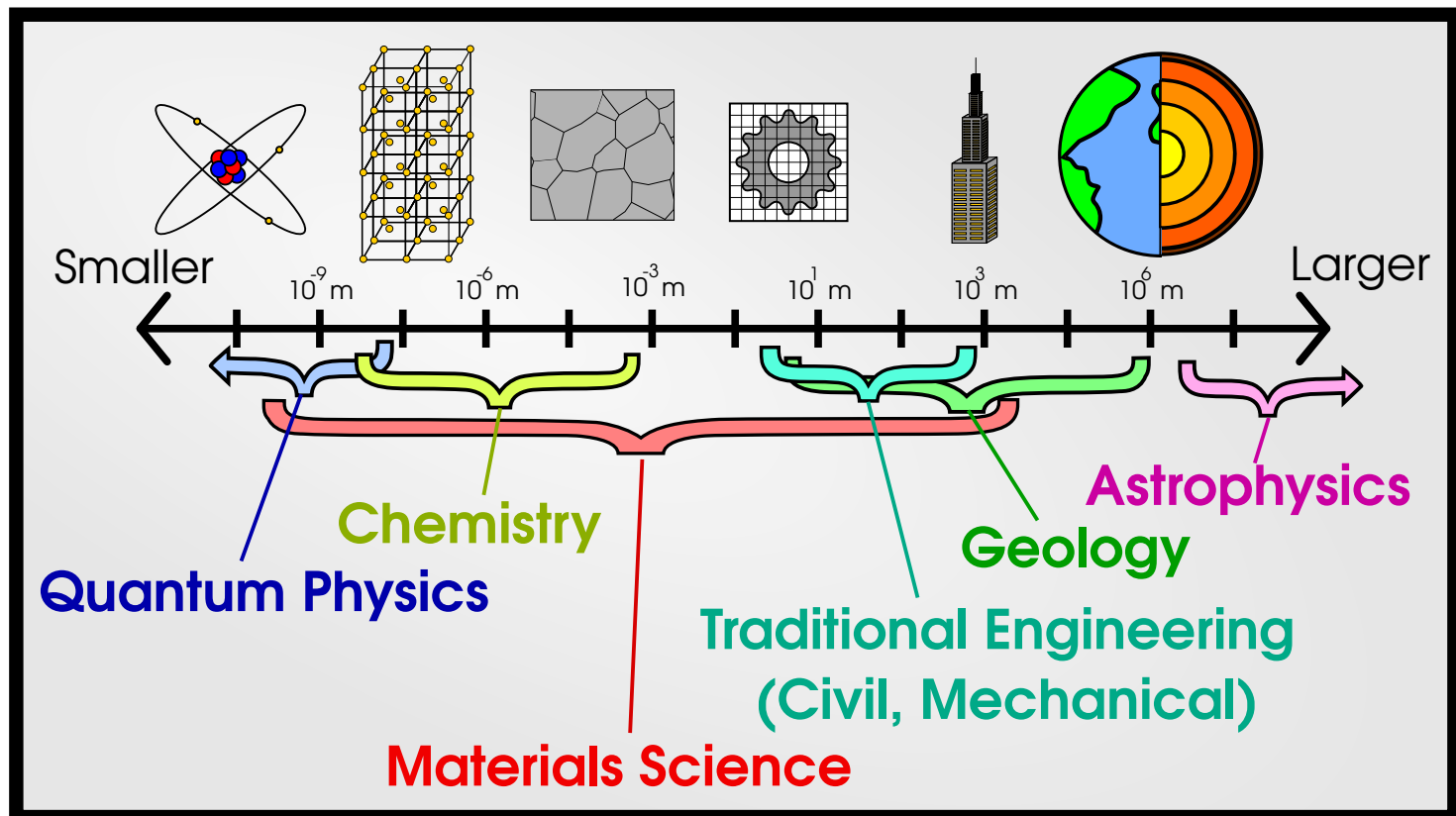
The band-gap problem in the modeling of insulators



Toward multi-scale physics

Can we go from small to big length scales?

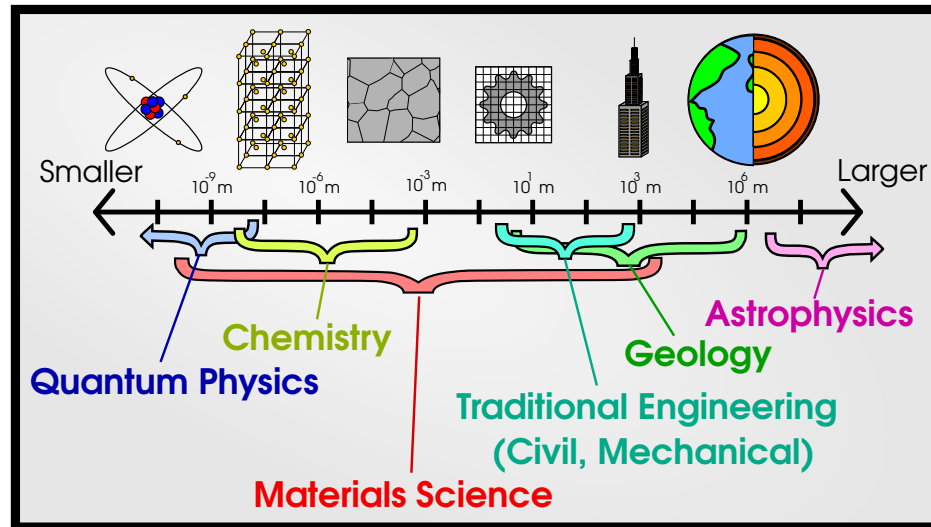
What if had infinite mathematical or computational power?



from mstudent.com

A final question

**How far can I go with a finite computational power?**



**Thank you for listening!**